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Card-based Cryptographic Protocols Using a Minimal Number of Cards

ASIACRYPT 2015 | Alexander Koch, Stefan Walzer, Kevin Härtel

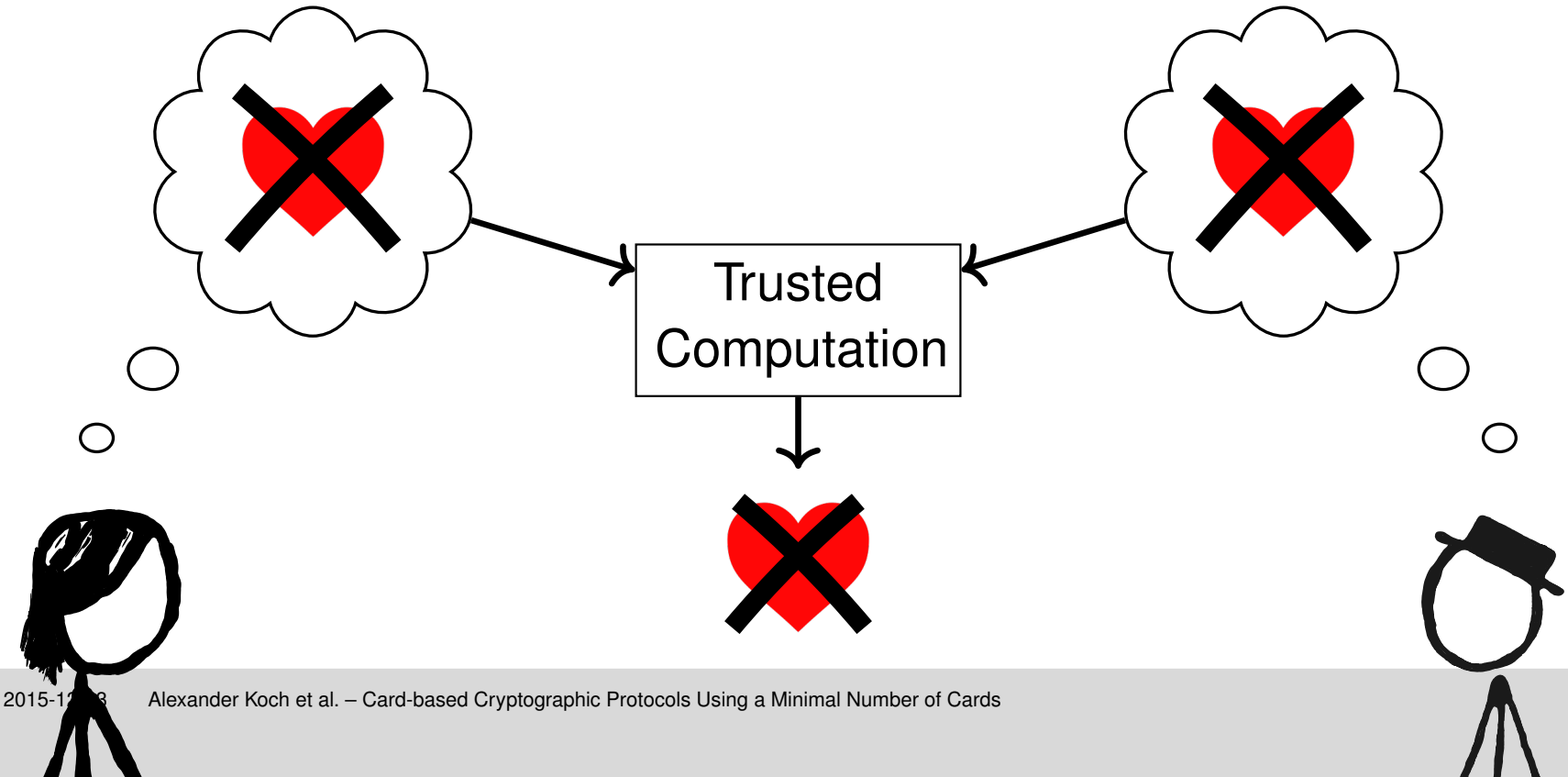
DEPARTMENT OF INFORMATICS, INSTITUTE OF THEORETICAL INFORMATICS

Motivating Scenario I

Calculating Mutual Interest with Playing Cards

- Secrets: Do I fancy him/her?
- To compute: Is there mutual interest?

⇝ Secure 2-party AND without computers



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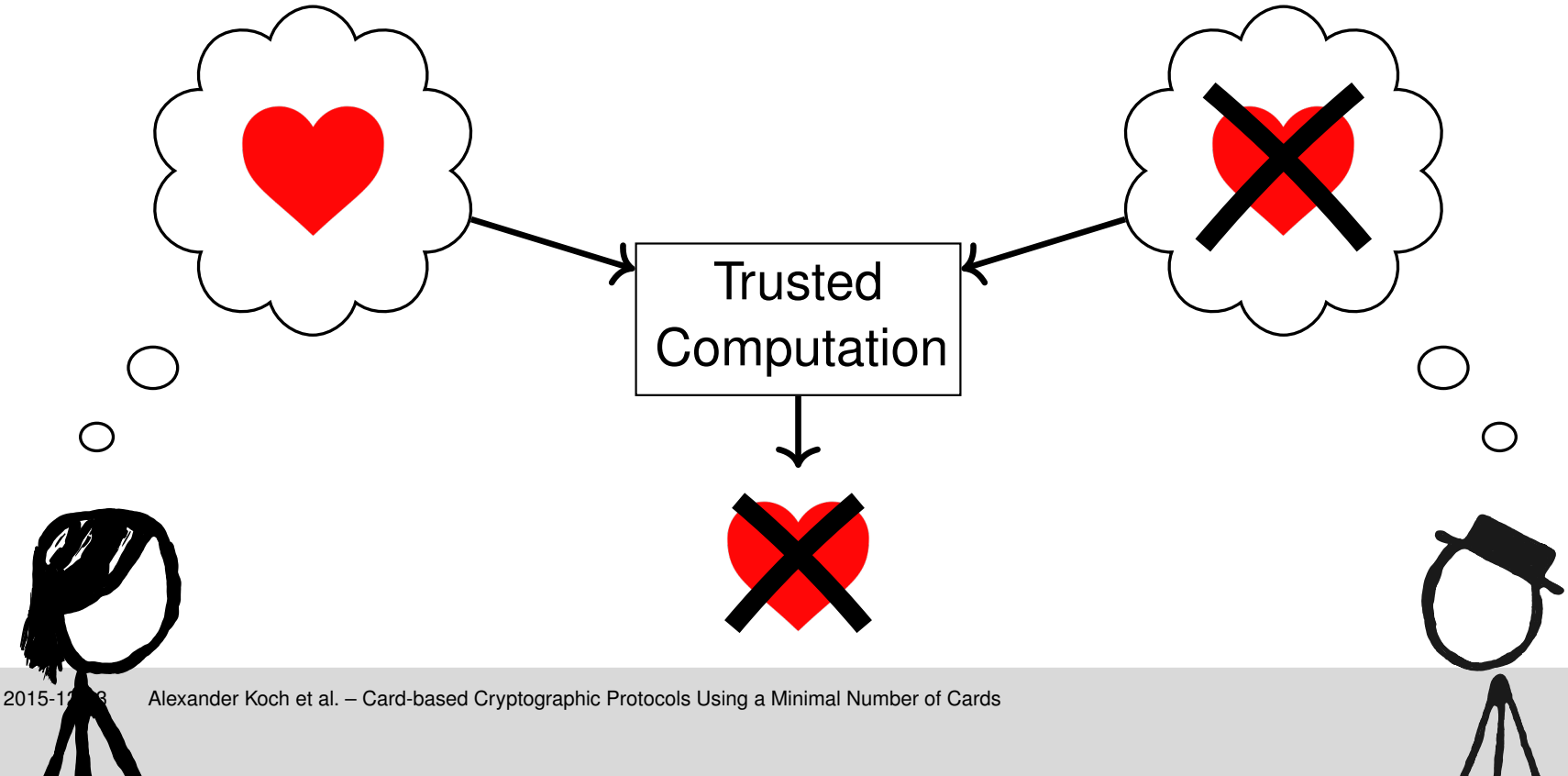

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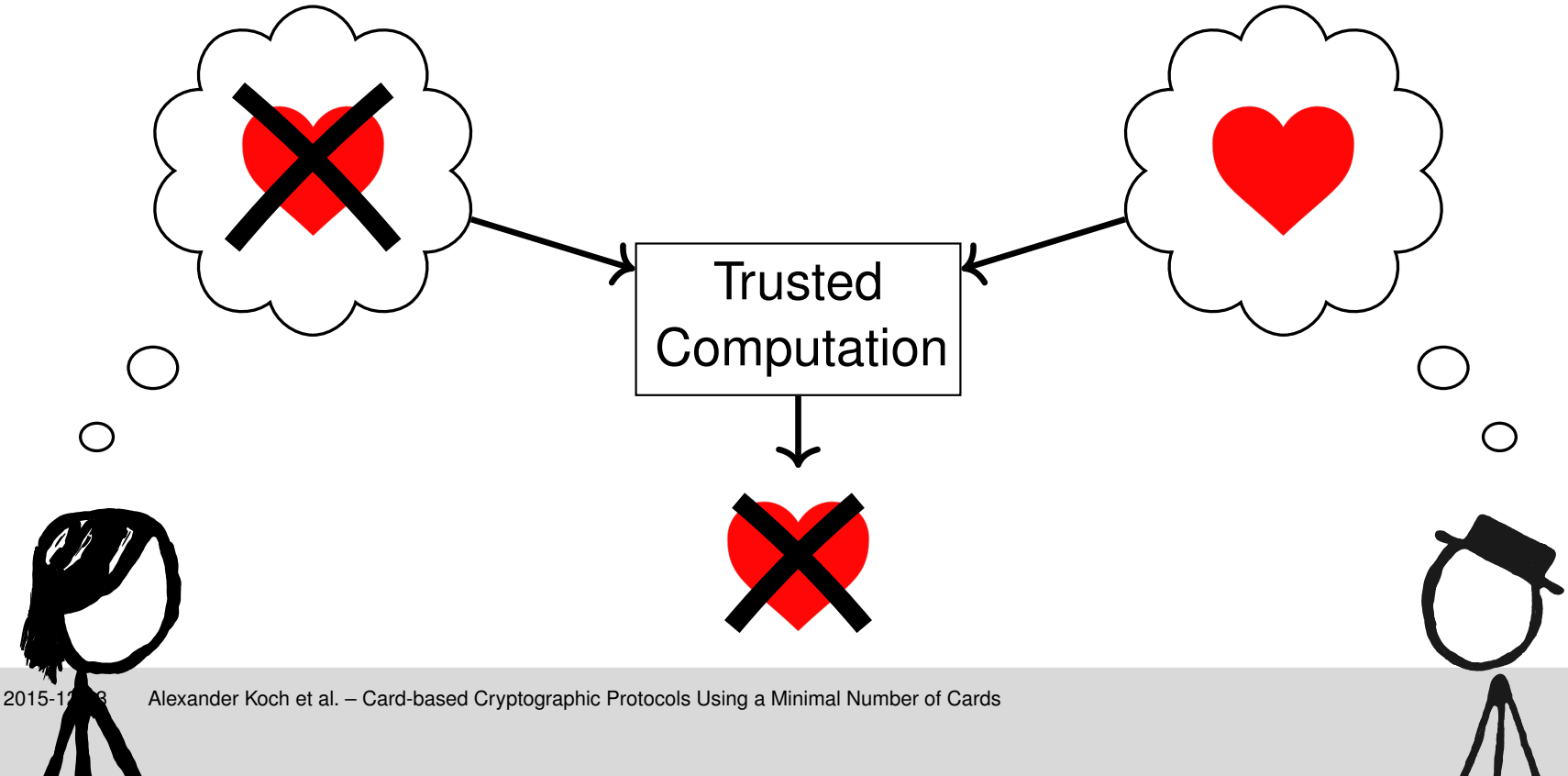

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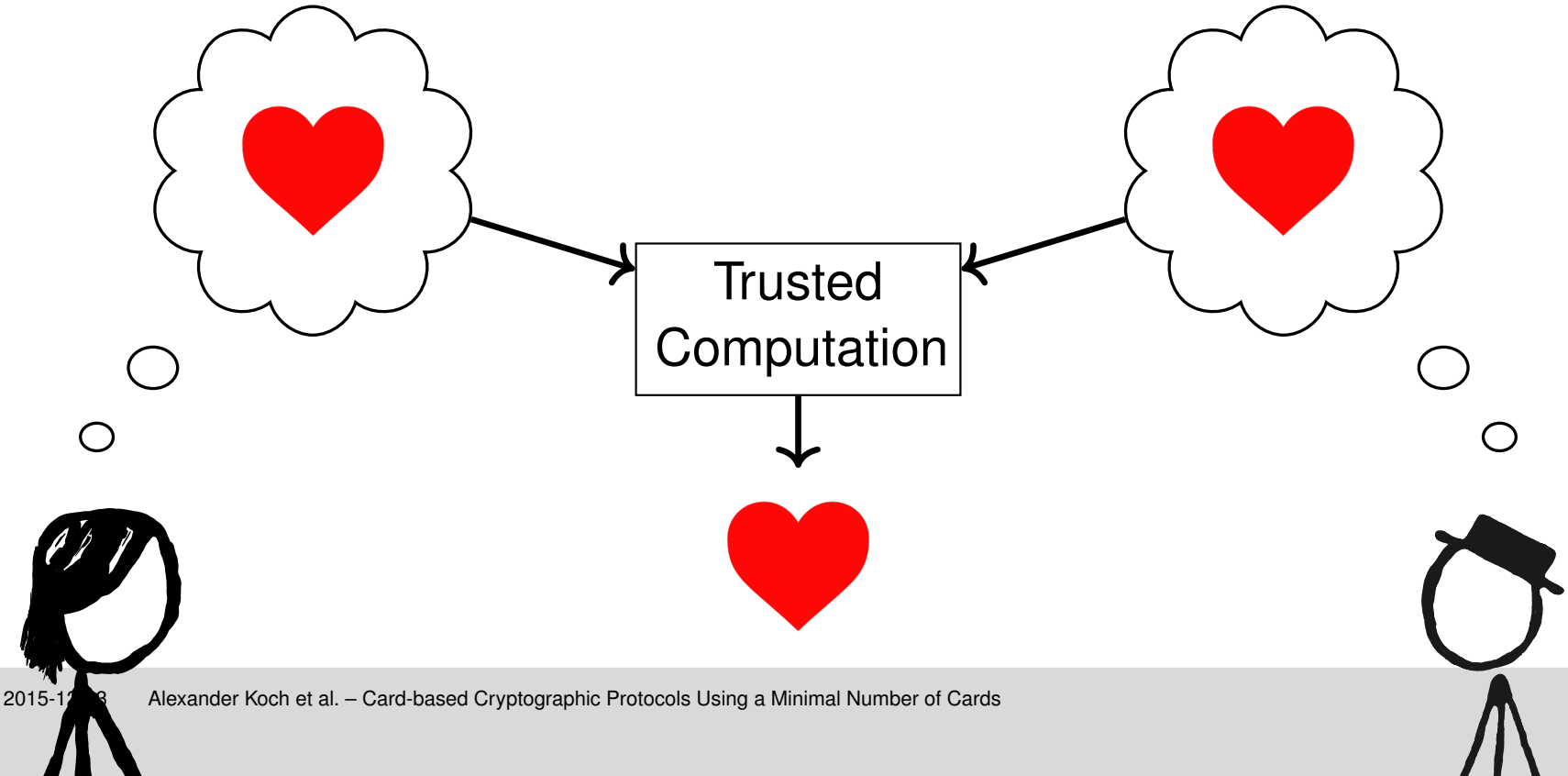

Motivating Scenario II

Explaining MPC to Non-Experts/Students

- You meet s.o. at a bar and want to explain MPC as an example from your work life.

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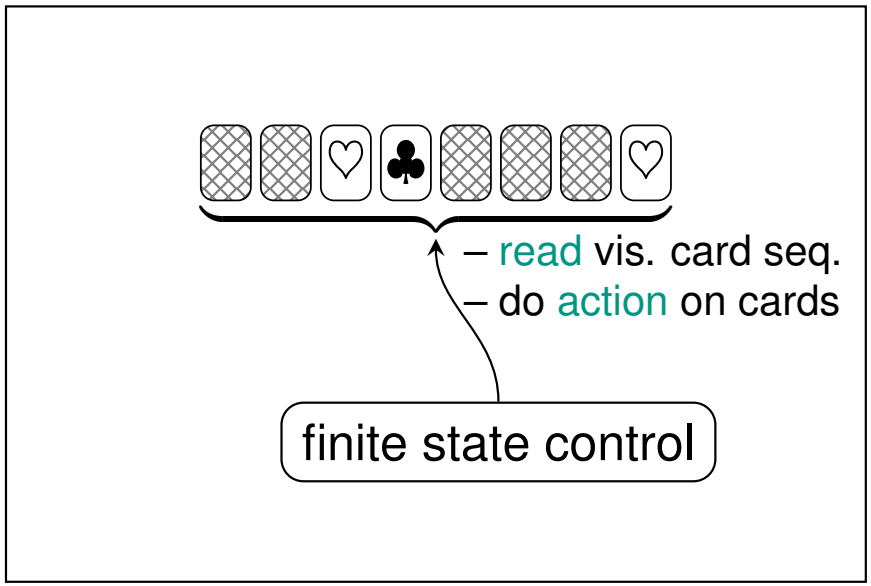
Motivating Scenario III

You are a theoretician

- What is possible with unconventional computational models?
- MPC from indistinguishability of cards & correct shuffling

⇝ cf. to physical assumptions like tamper-proofness of hardware



Setting and Goal

Two types of indistinguishable cards:

Heart ♡ and club ♣ with backside ▨.

Encode bits as

- ♣ ♡ $\hat{=}$ 0
- ♡ ♣ $\hat{=}$ 1

Our goal (“committed format”)

- Take face-down input ▨ ▨ ▨ ▨ (bits a, b)
- Compute face-down output ▨ ▨ ($a \wedge b$)
- Learn nothing about the input or output during protocol run.

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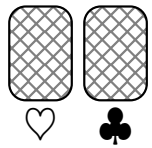

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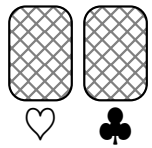

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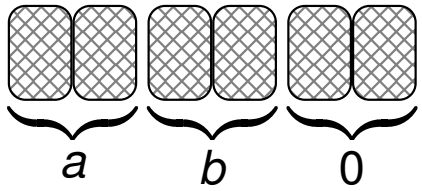
A Simple Six-Card AND Protocol

Mizuki and Sone [MS09]

Observation: $(a \wedge b) \equiv (\text{if} \quad a \text{ then } b \text{ else } 0)$

The Protocol:



- Turn first two cards

$\rightsquigarrow$ result is cards 3, 4

$\rightsquigarrow$ result is cards 5, 6

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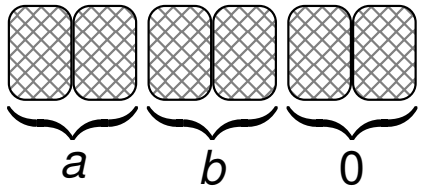

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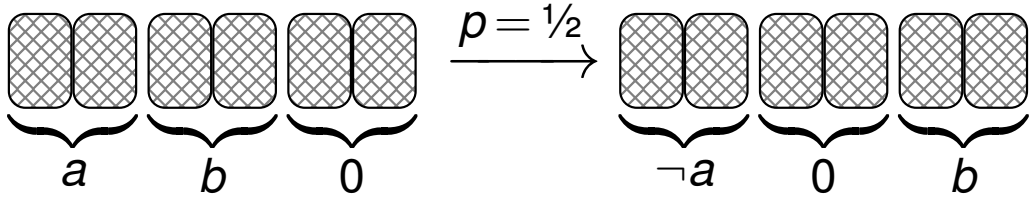

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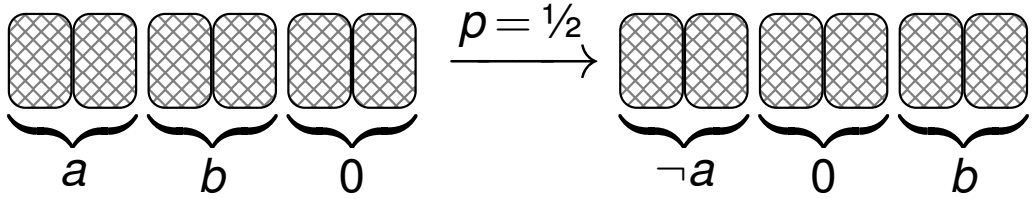

- With probability ½: Apply permutation $(1\ 2)(3\ 5)(4\ 6)$.
- For privacy: each player once, without the other looking.
- Turn first two cards


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Demonstration of the Six-Card-Protocol

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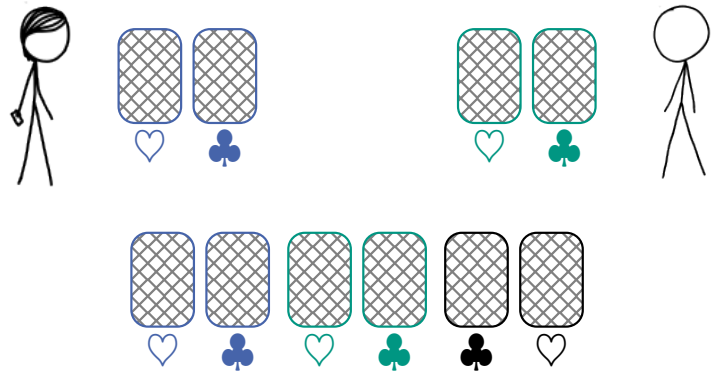

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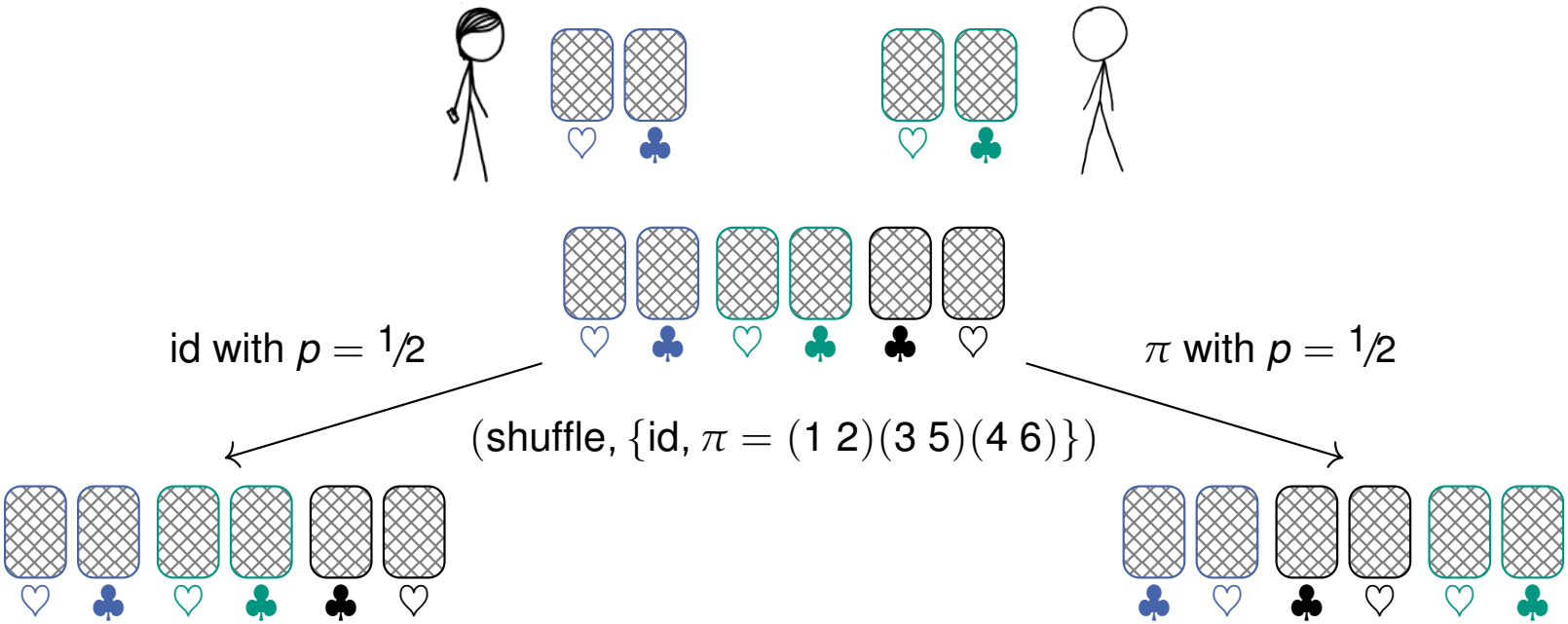

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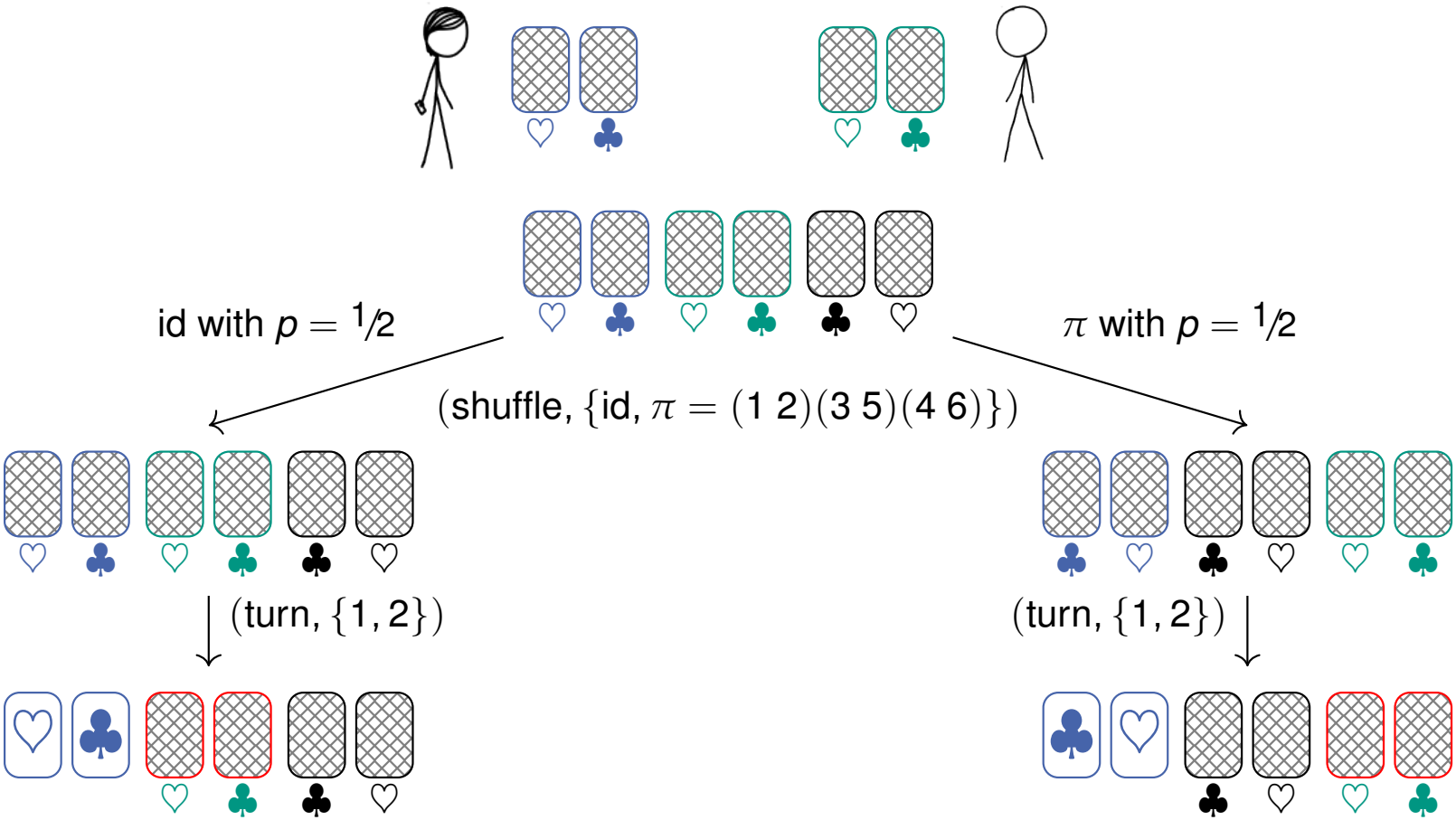

Can we do better than six cards?

Open problem from [MS09; MS14; MKS12]

Main Question: Can $a \wedge b$ be computed with 4 cards?
in committed format (in the model of Mizuki and Shizuya [MS14])

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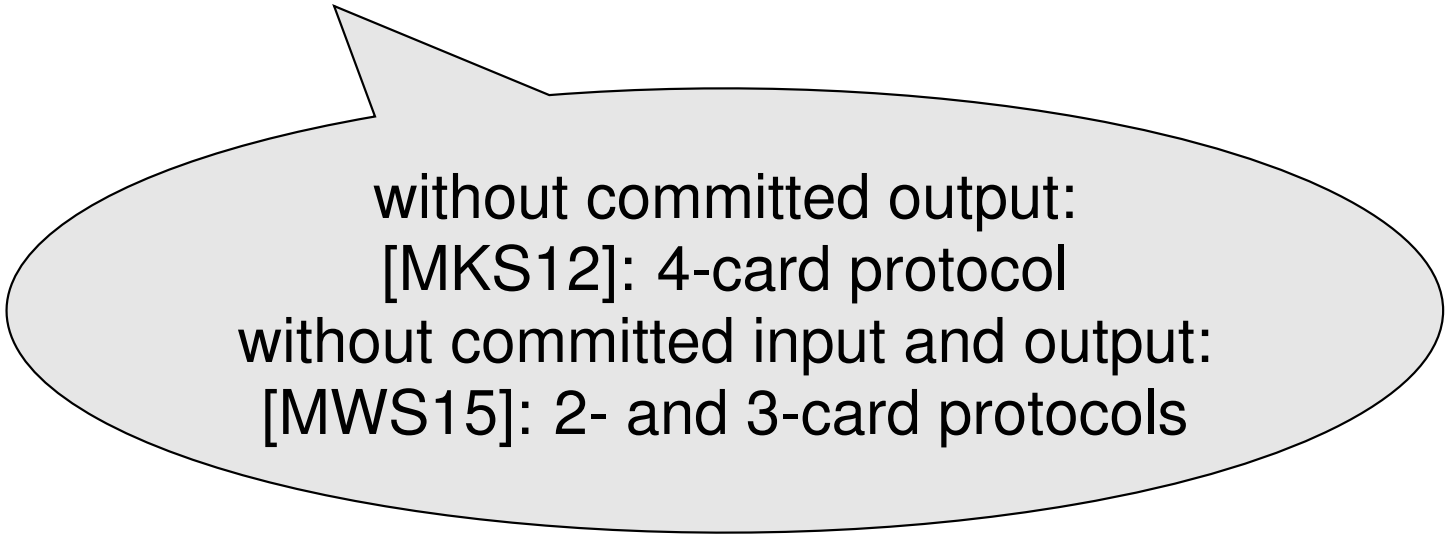

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Our Results

1. Yes, 4 cards suffice...
2. But 4-card protocols are necessarily Las Vegas (LV)
   - no a priori bound on runtime
   - method: analyze "states" of protocols
3. Yes, 5 cards suffice for finite-runtime protocols
4. LV protocol for k-ary functions using $2k$ cards
5. Note: "Complex" Shuffles needed.

State Transitions: The Six-Card Protocol

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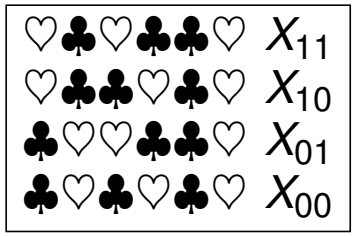
Protocol State:

Annotate currently possible sequences with probability in terms of symbolic input prob.

$X_{ij} = \Pr[a = i, b = j]$



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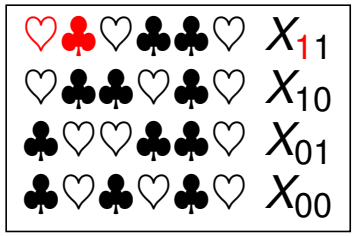

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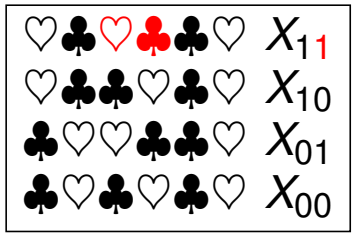

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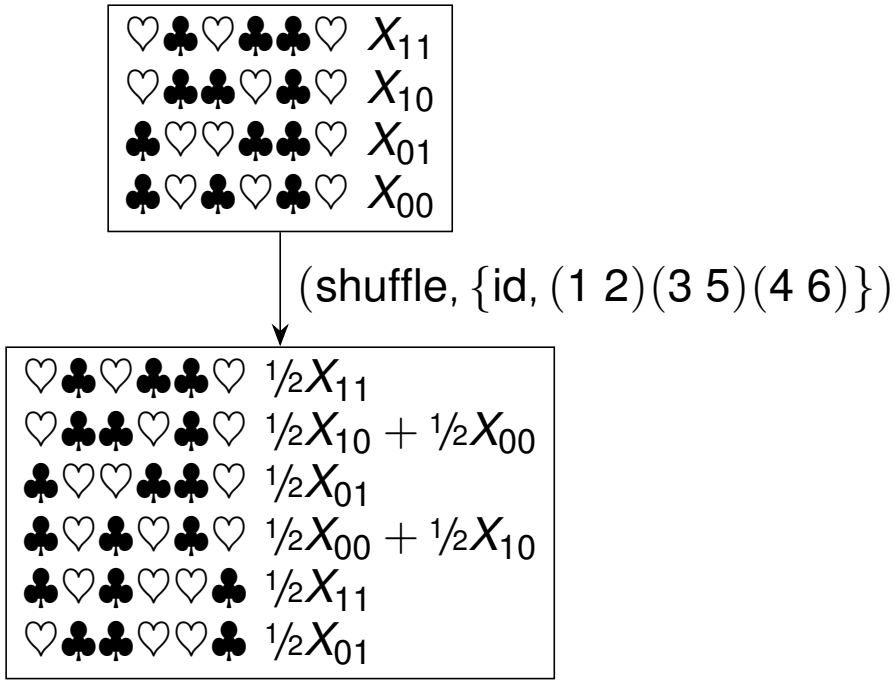

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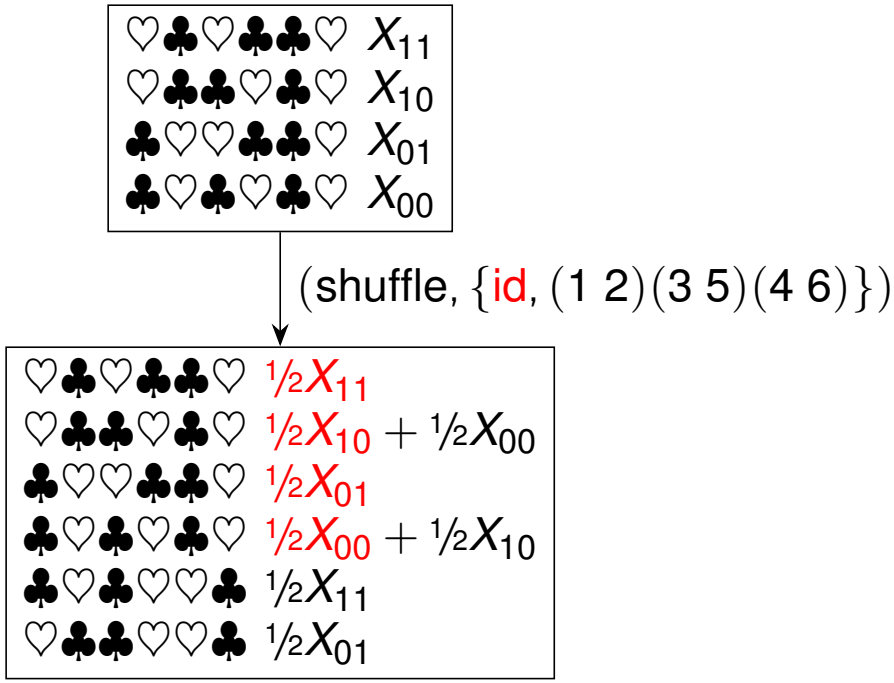

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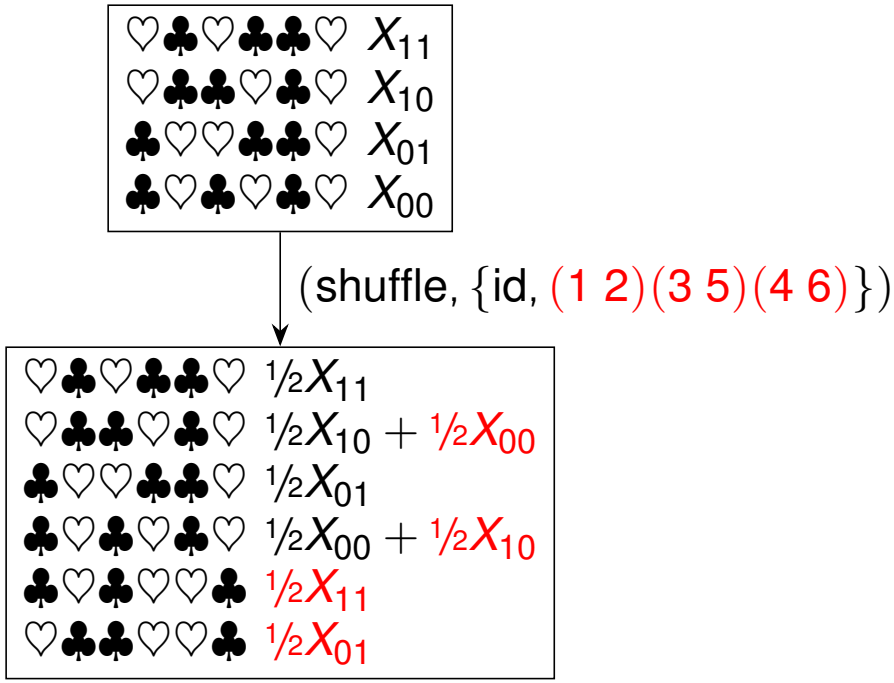

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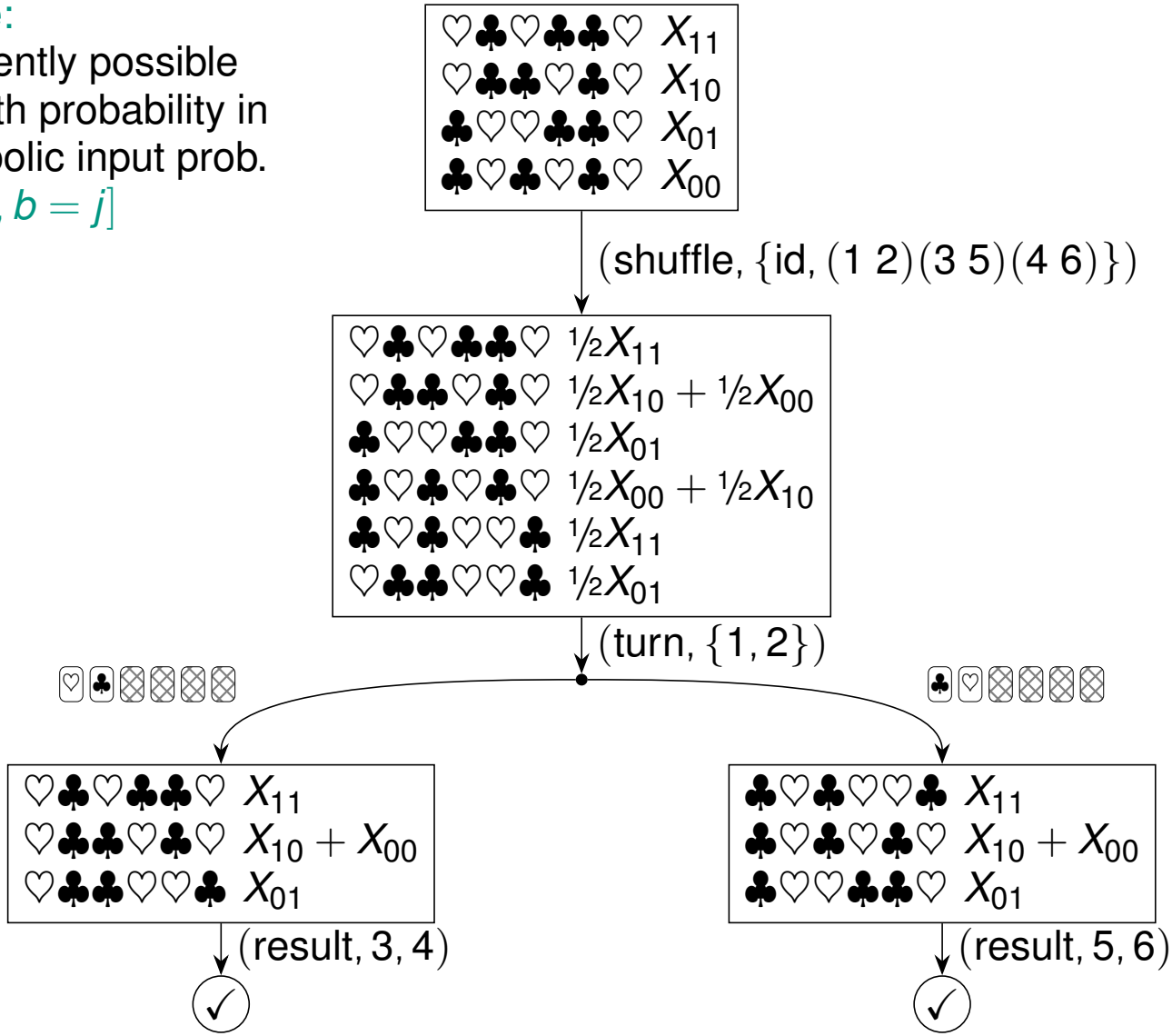

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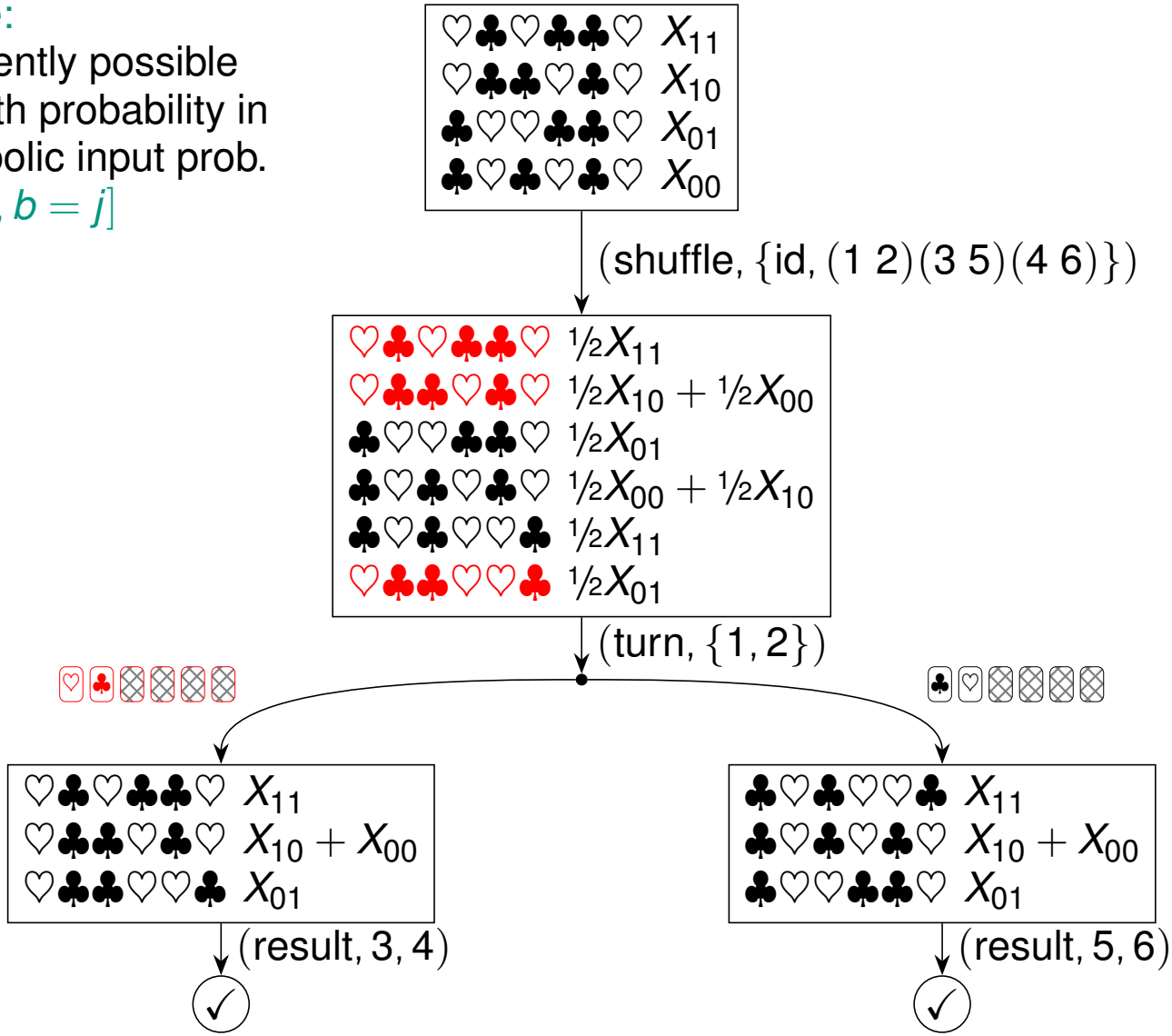

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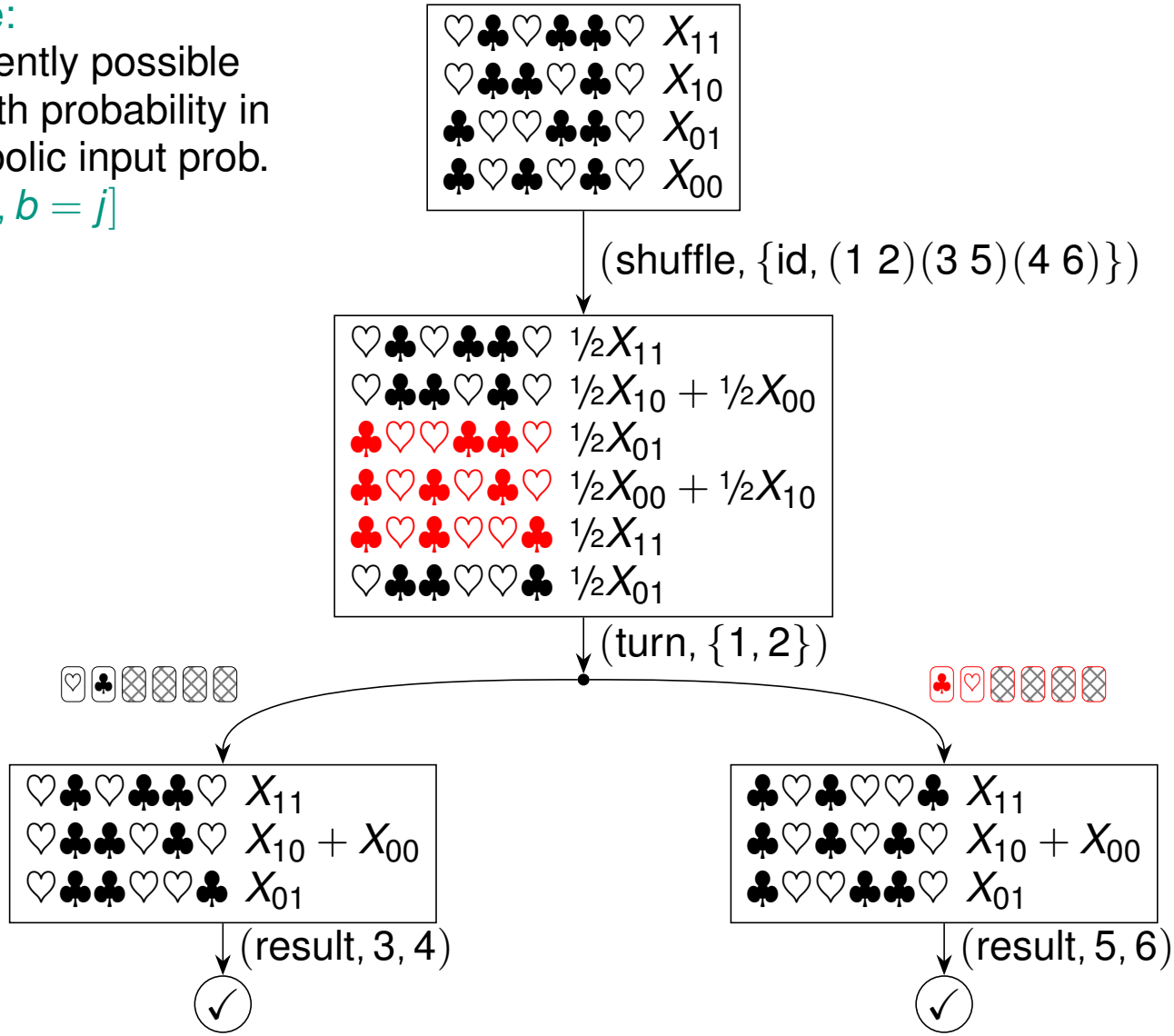

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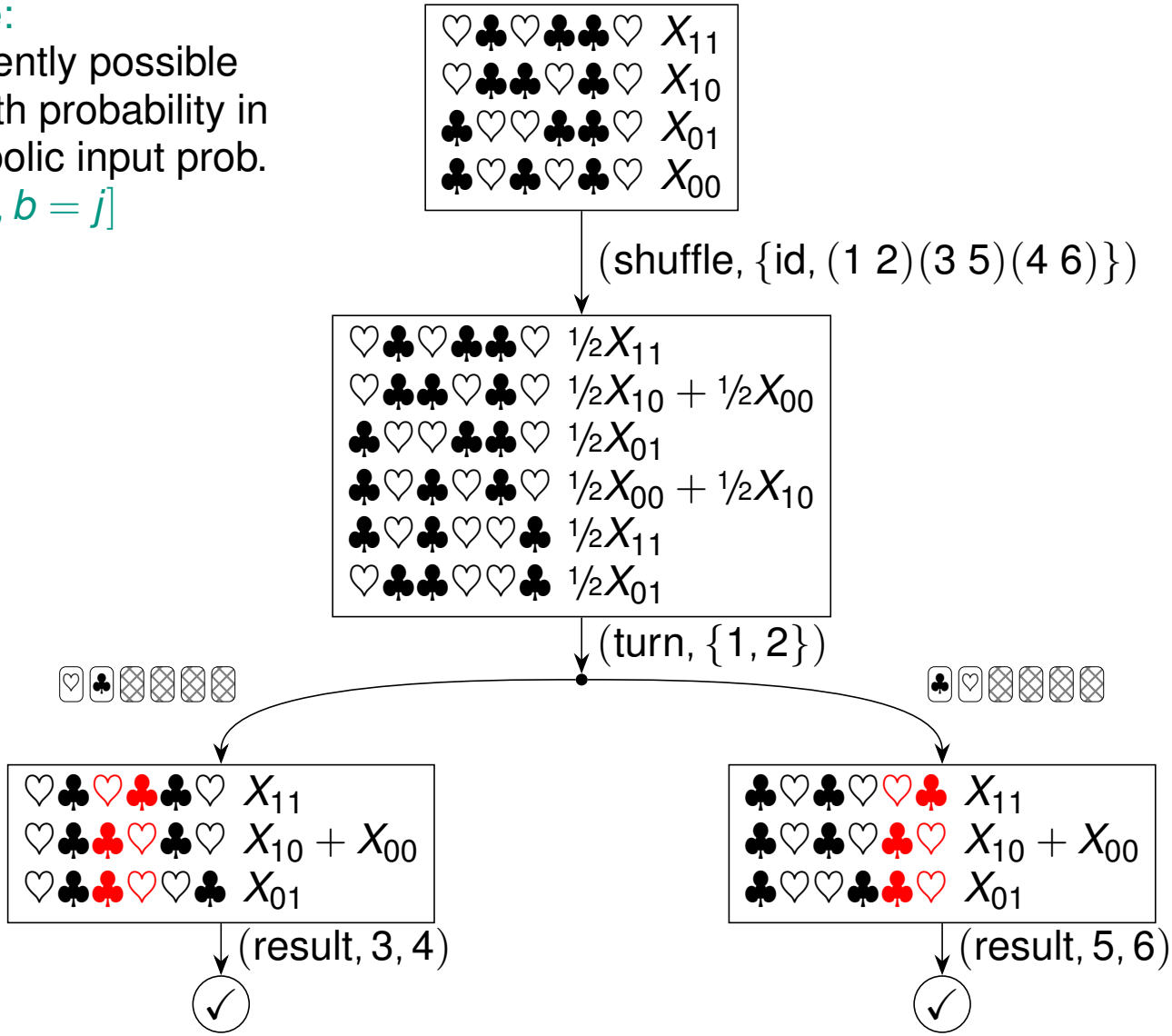

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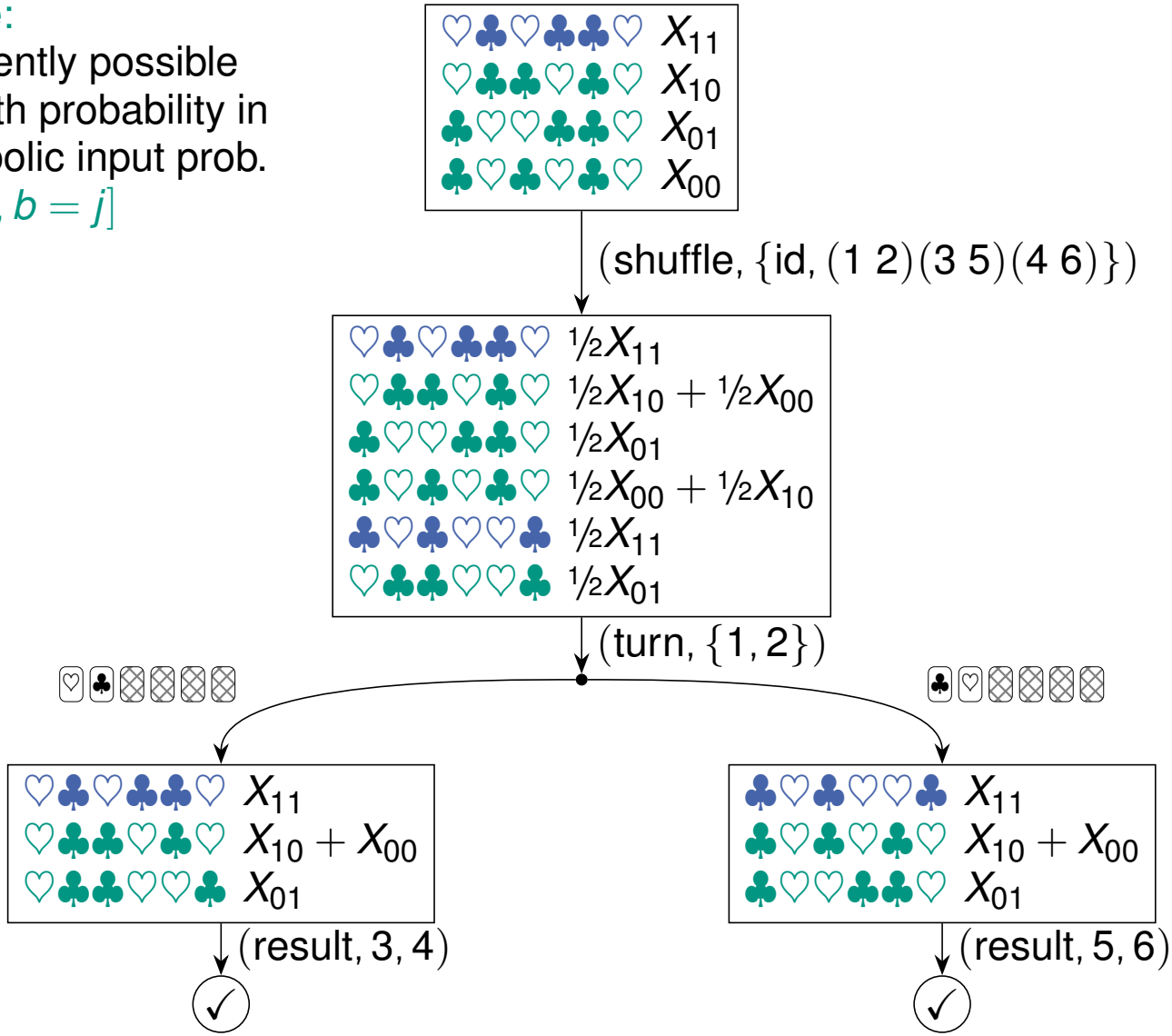


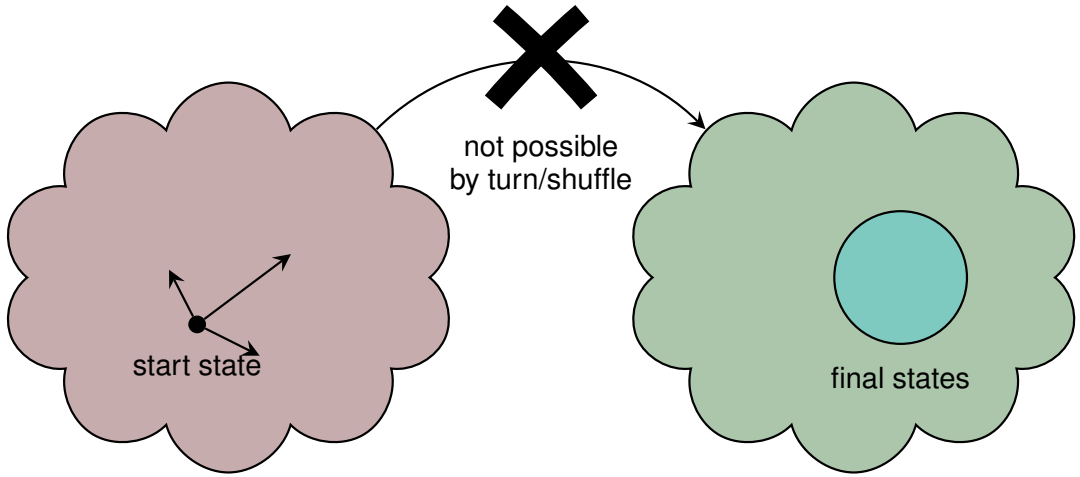
Impossibility Result

Theorem

There is no secure finite-runtime four-card AND protocol

Proof Idea

- Each sequence belongs either to output 0 or to 1.
- An $i|j$-state has i 0-sequences and j 1-sequences.
- Define non-reachable “good” states:



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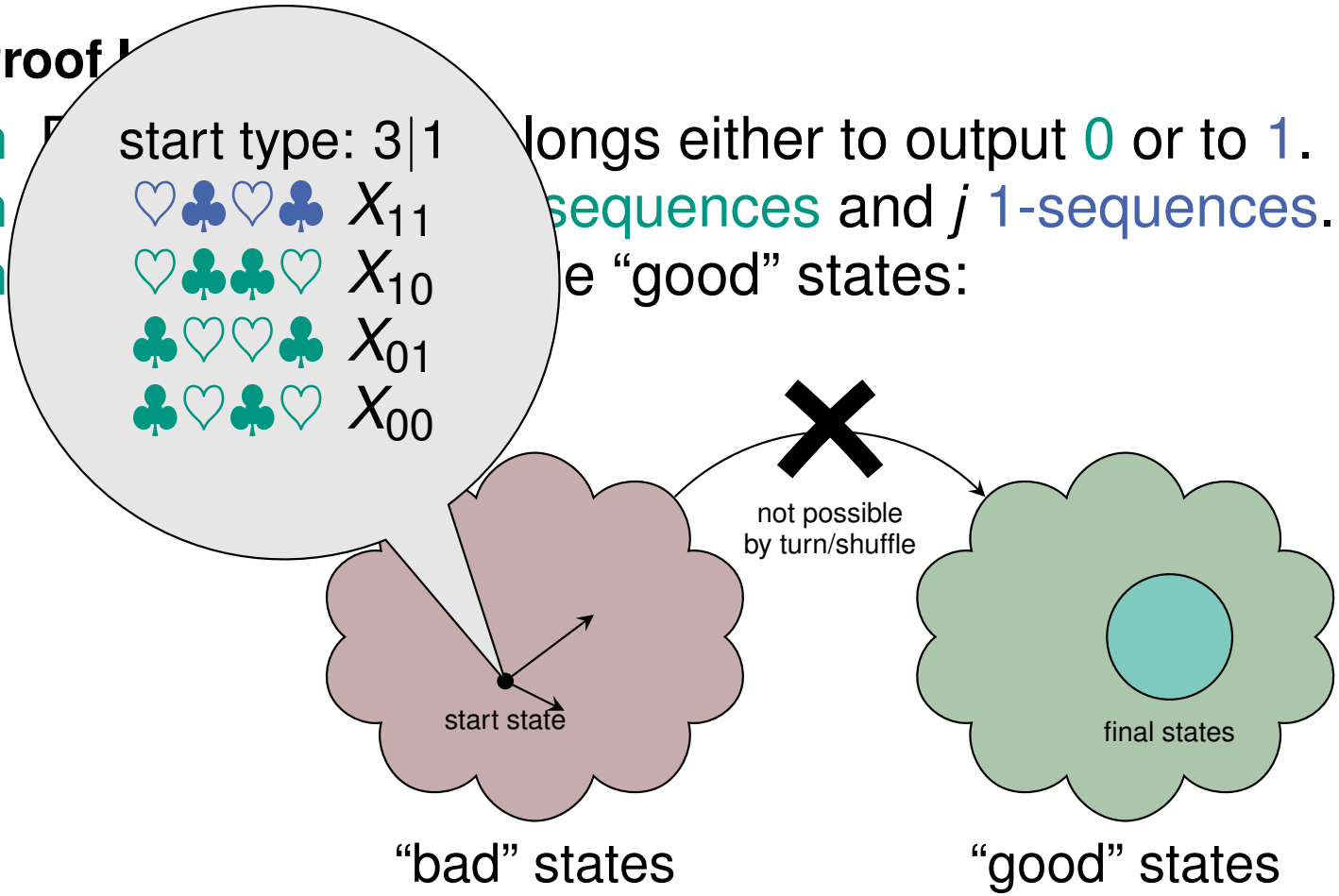

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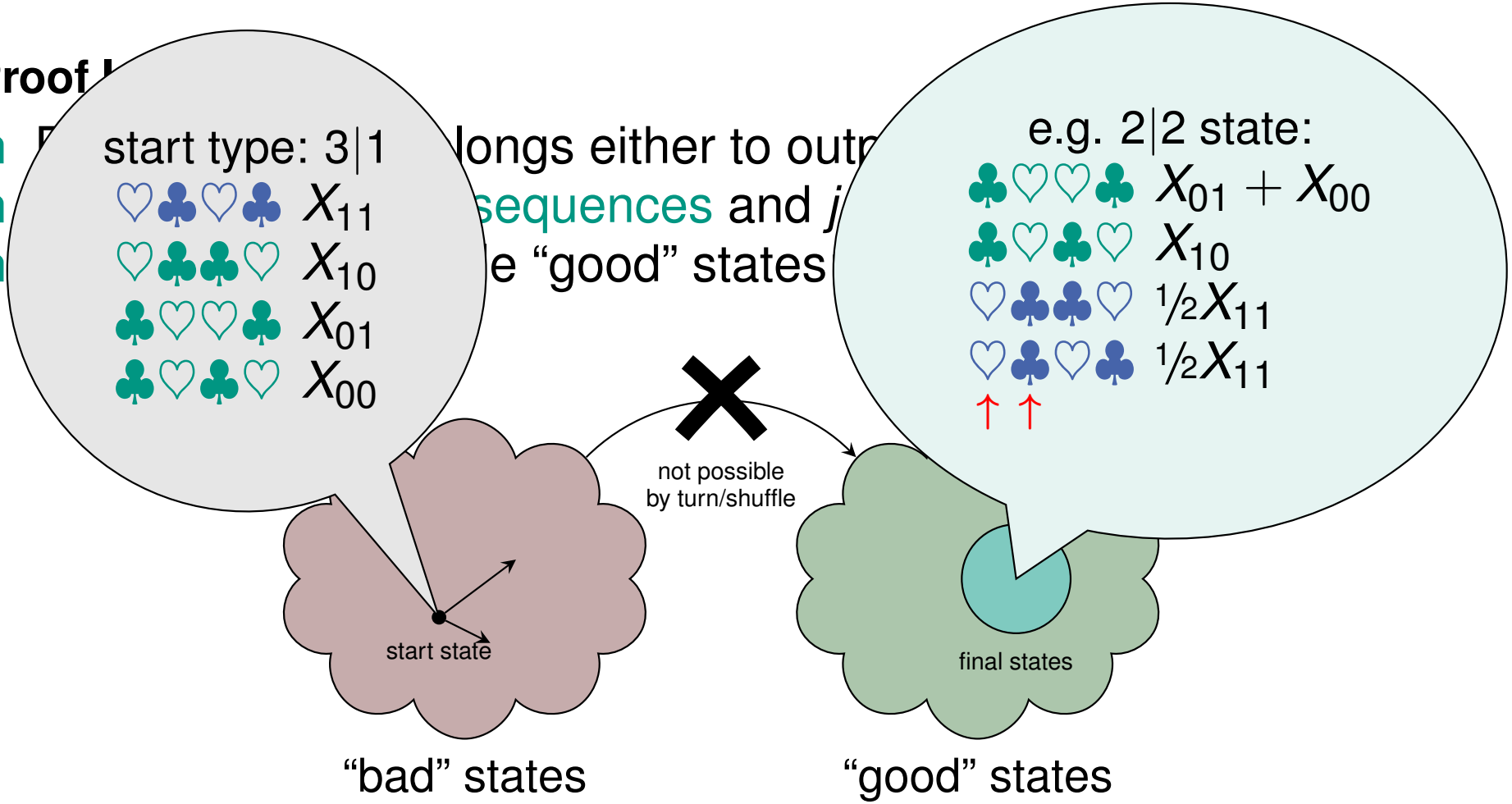
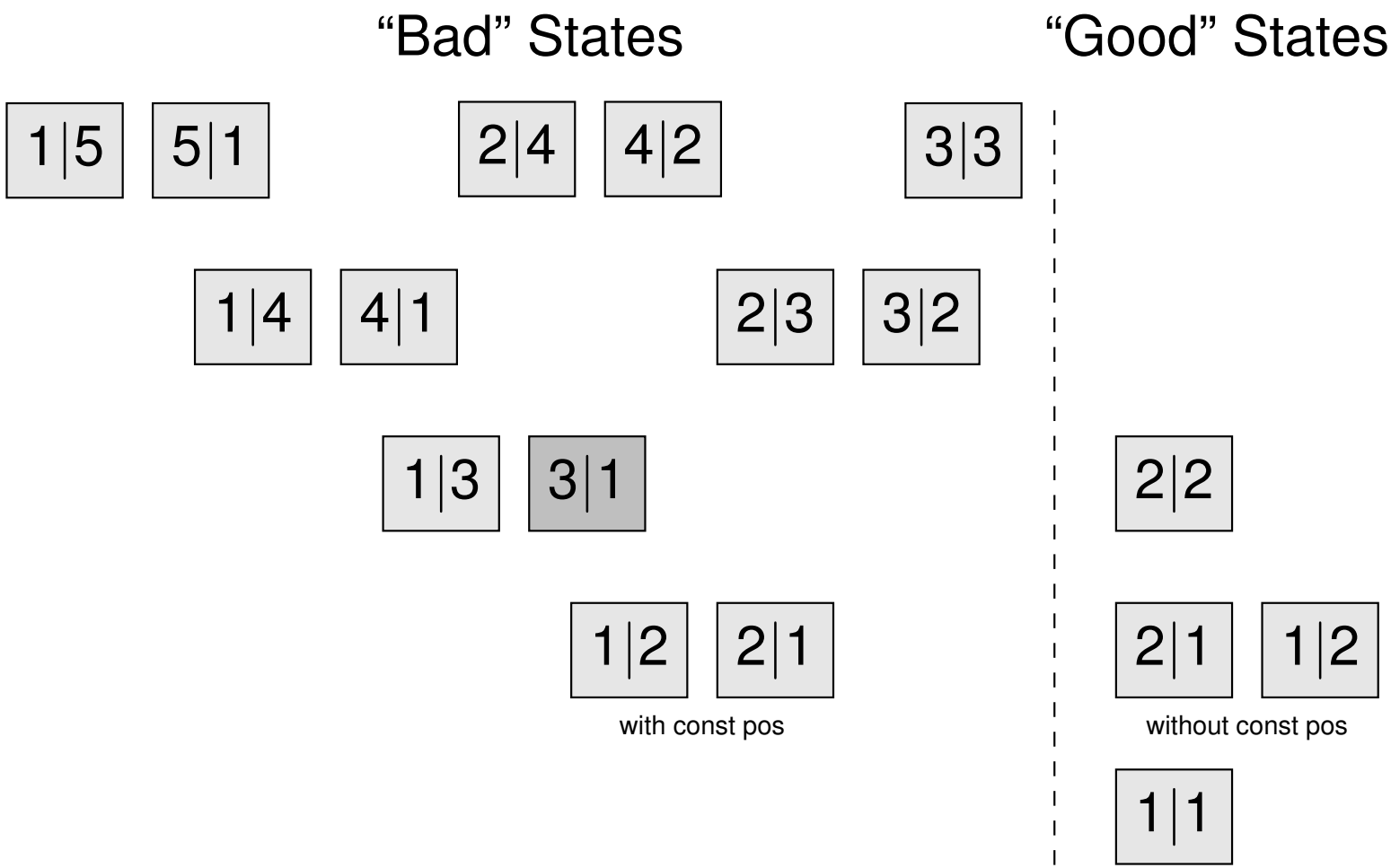


Proof Idea – Single Card Turns



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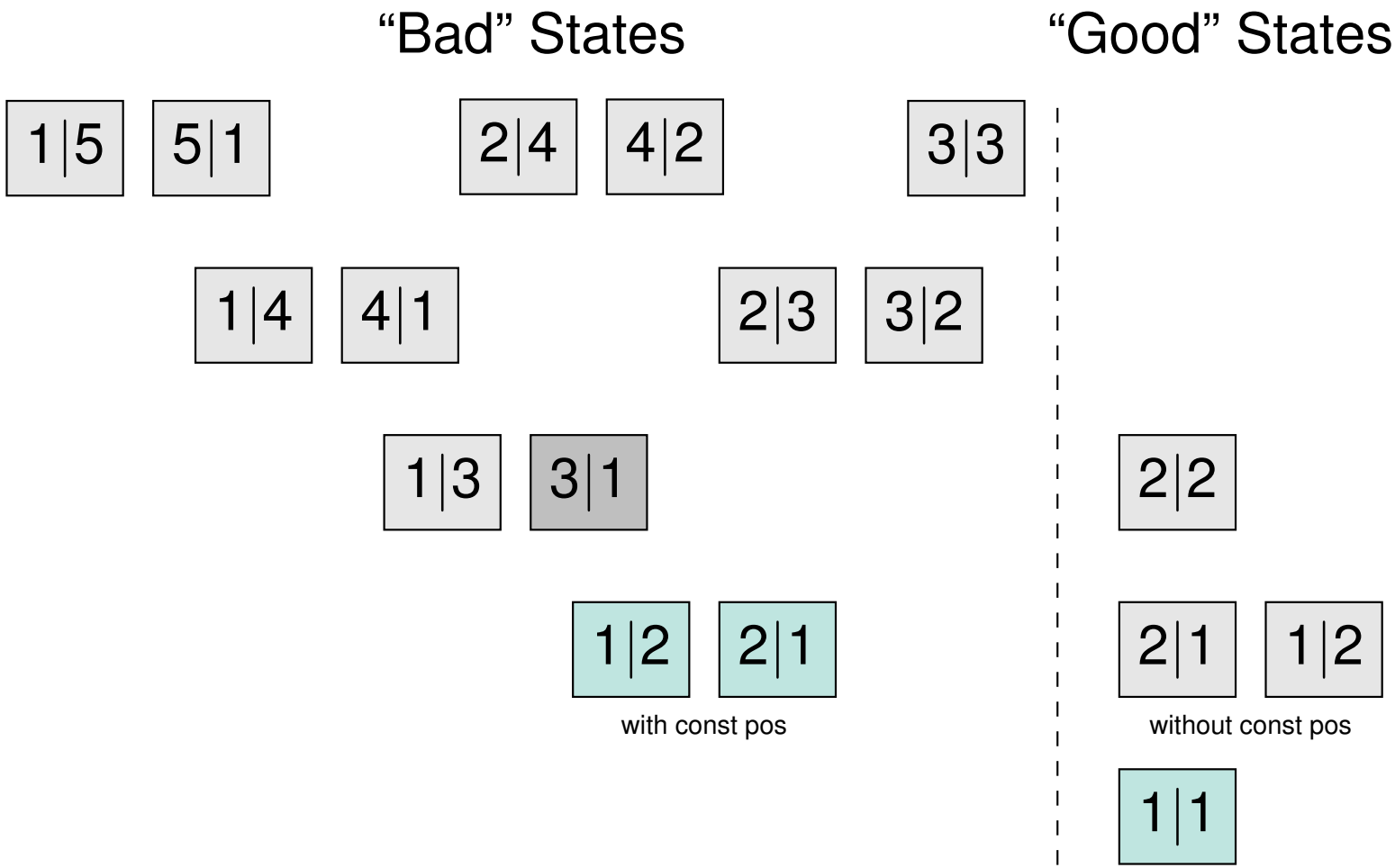


Observation 1. After turn: with const pos. and ≤ 3 sequences.

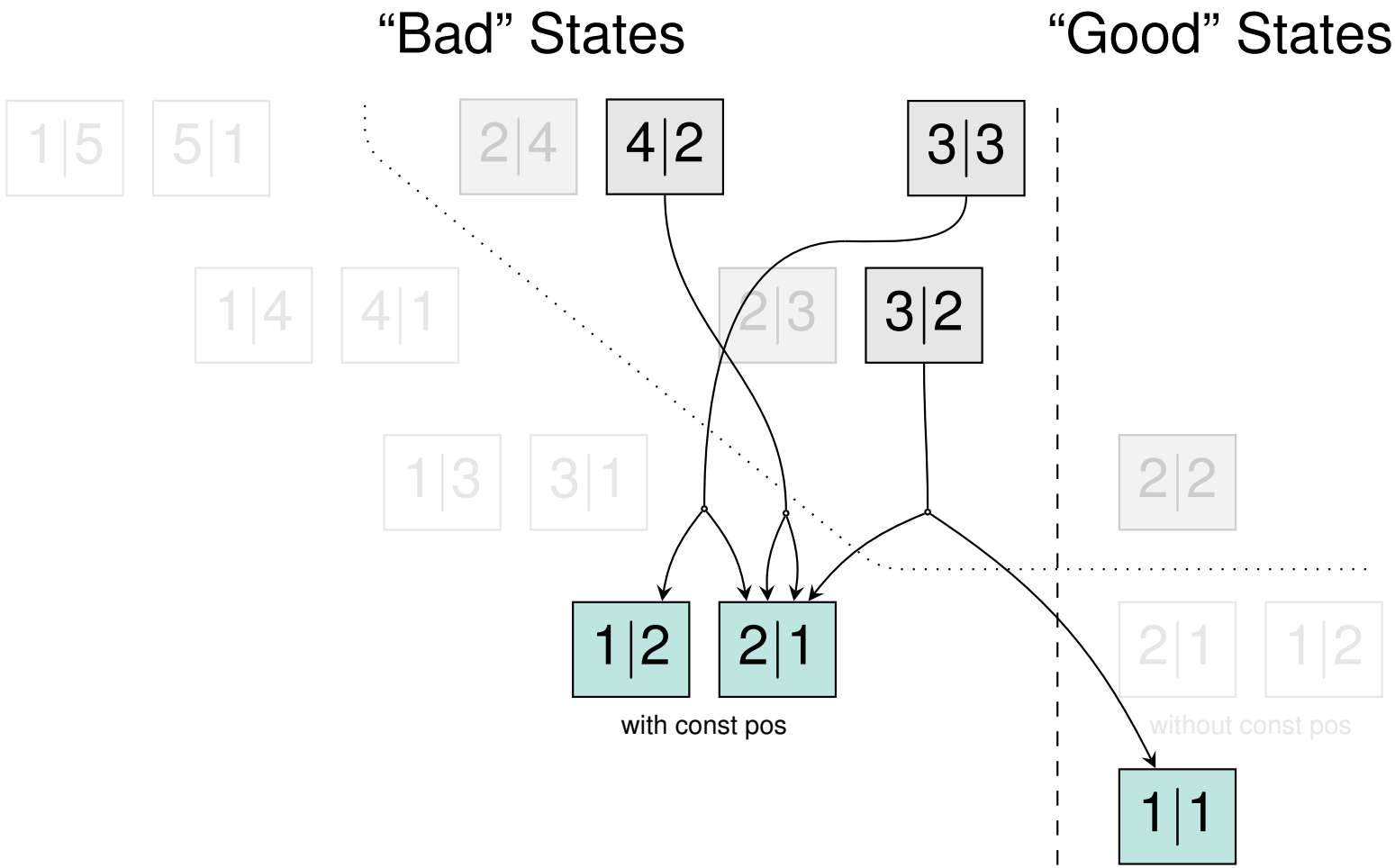
Proof Idea – Single Card Turns

Observation 2. Turnable states are $i|j$ with $i, j \geq 2$.

Proof Idea – Single Card Turns

Observation 3. W.l.o.g. we need to consider half of the states.

Proof Idea – Single Card Turns



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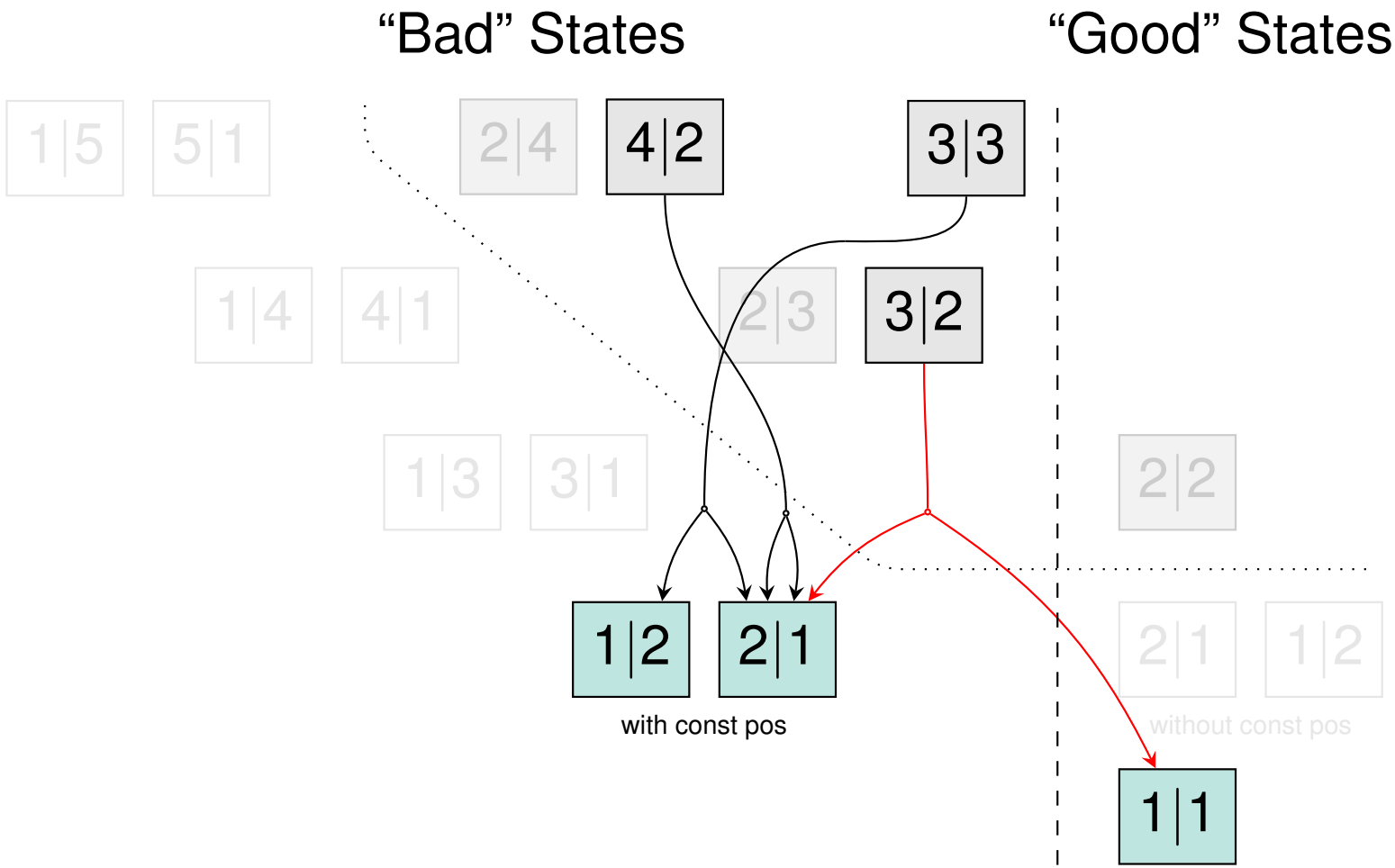
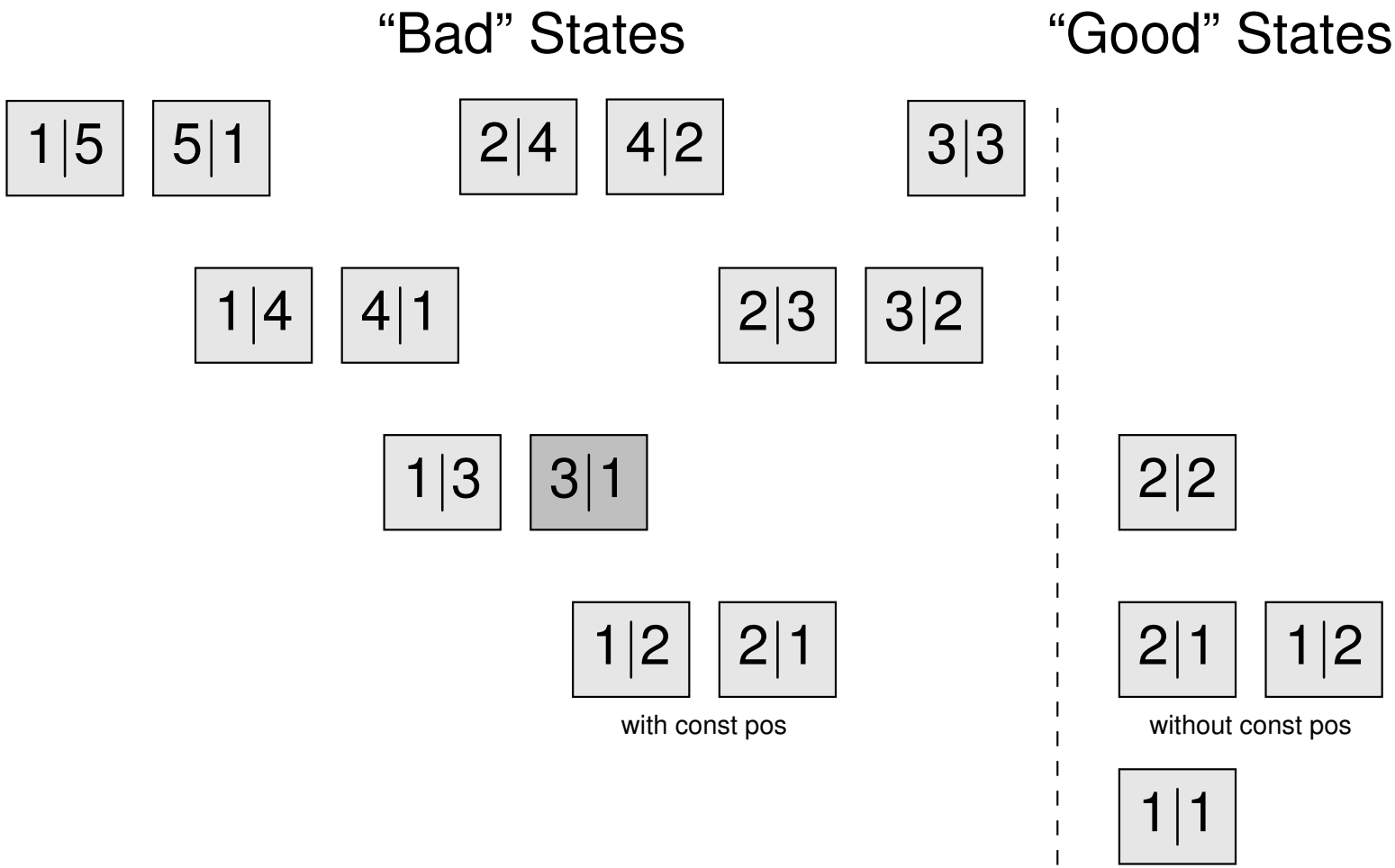


Proof Idea – Shuffles



Observation 1. Shuffles increase #sequences per type

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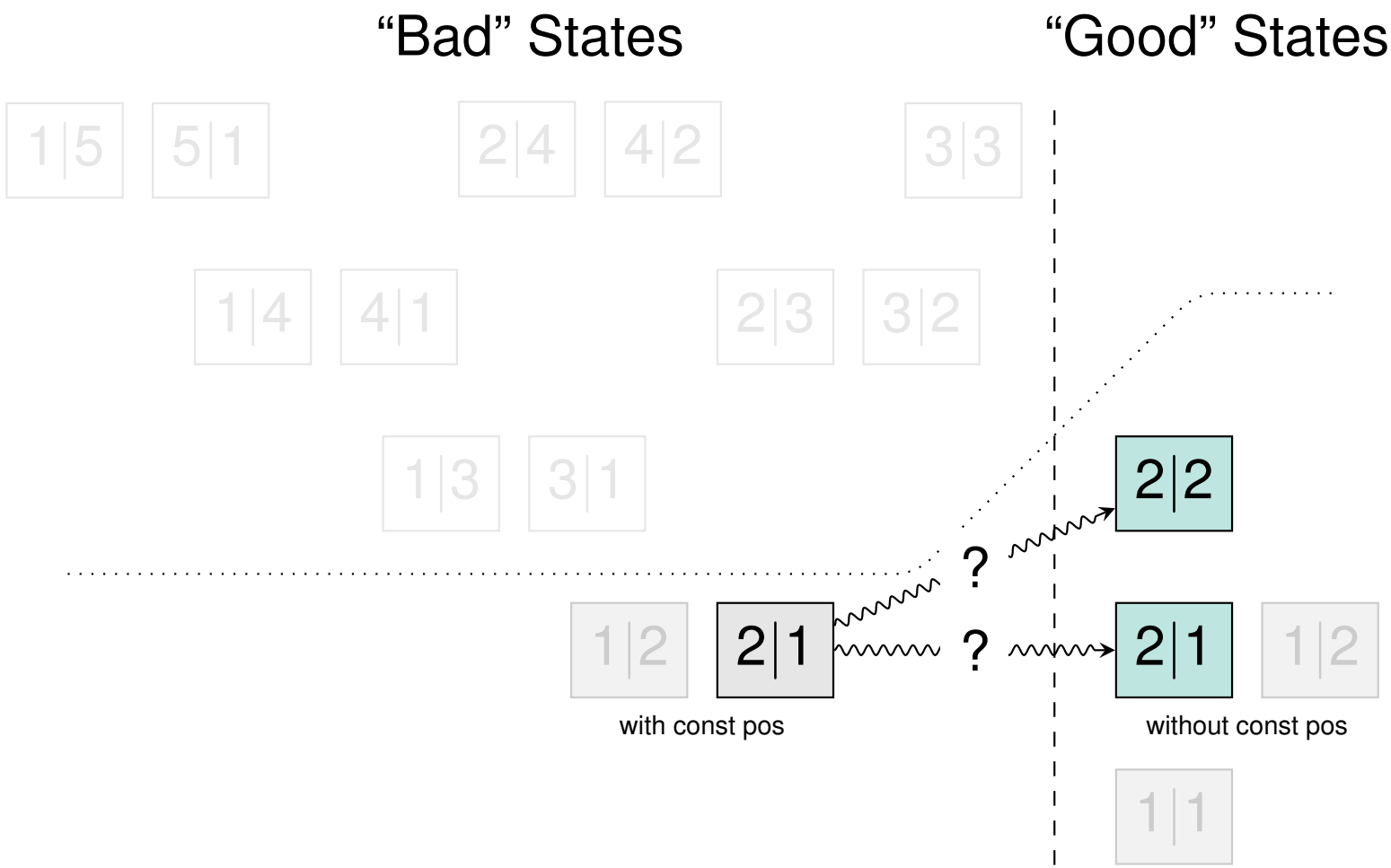

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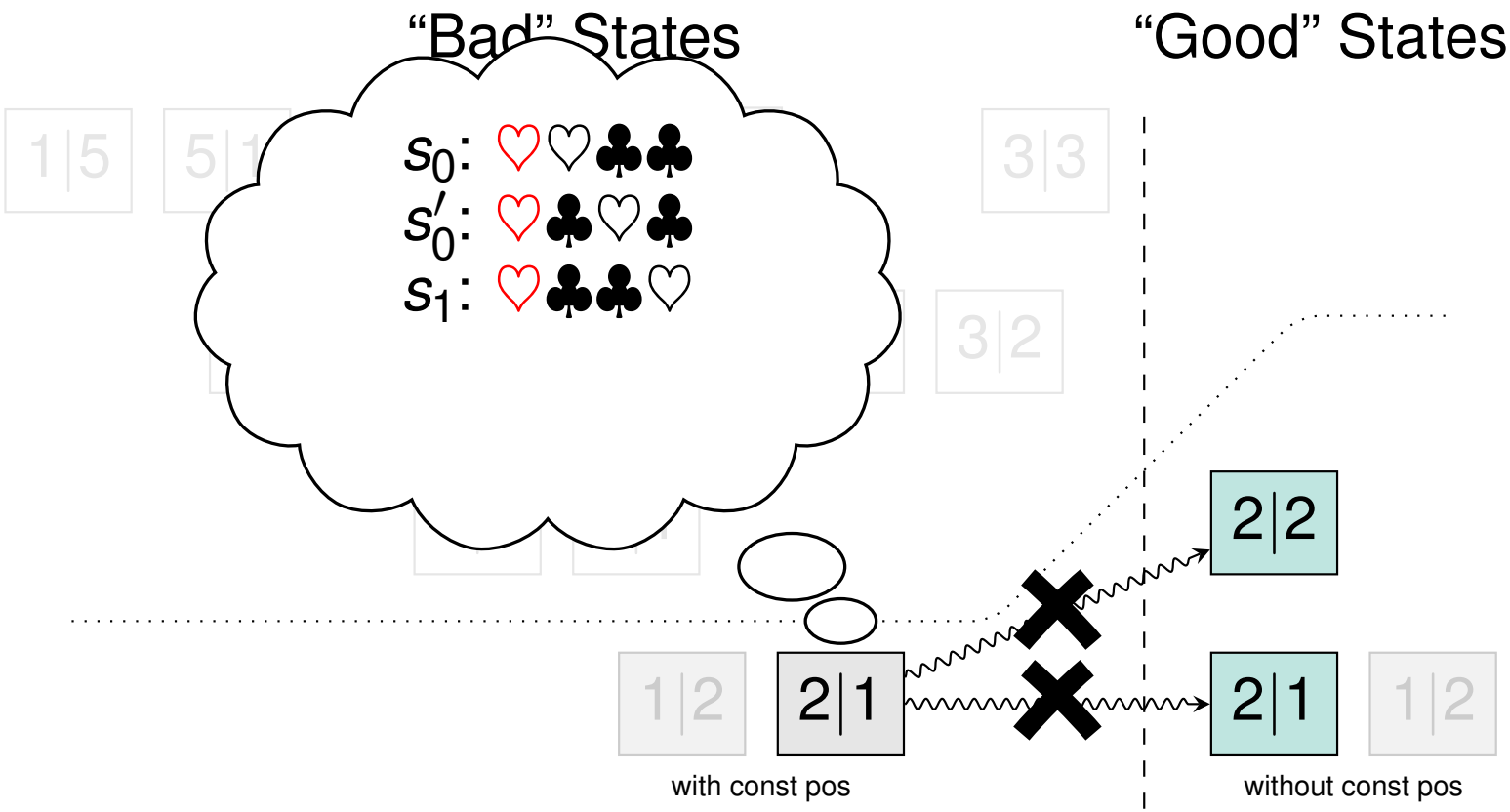

Apply (shuffle, Π, $\mathcal{F}$) to this state.
Case 1: All $\pi \in \Pi$ put constant column to same position.
$\Longrightarrow$ the resulting state still has a constant column.

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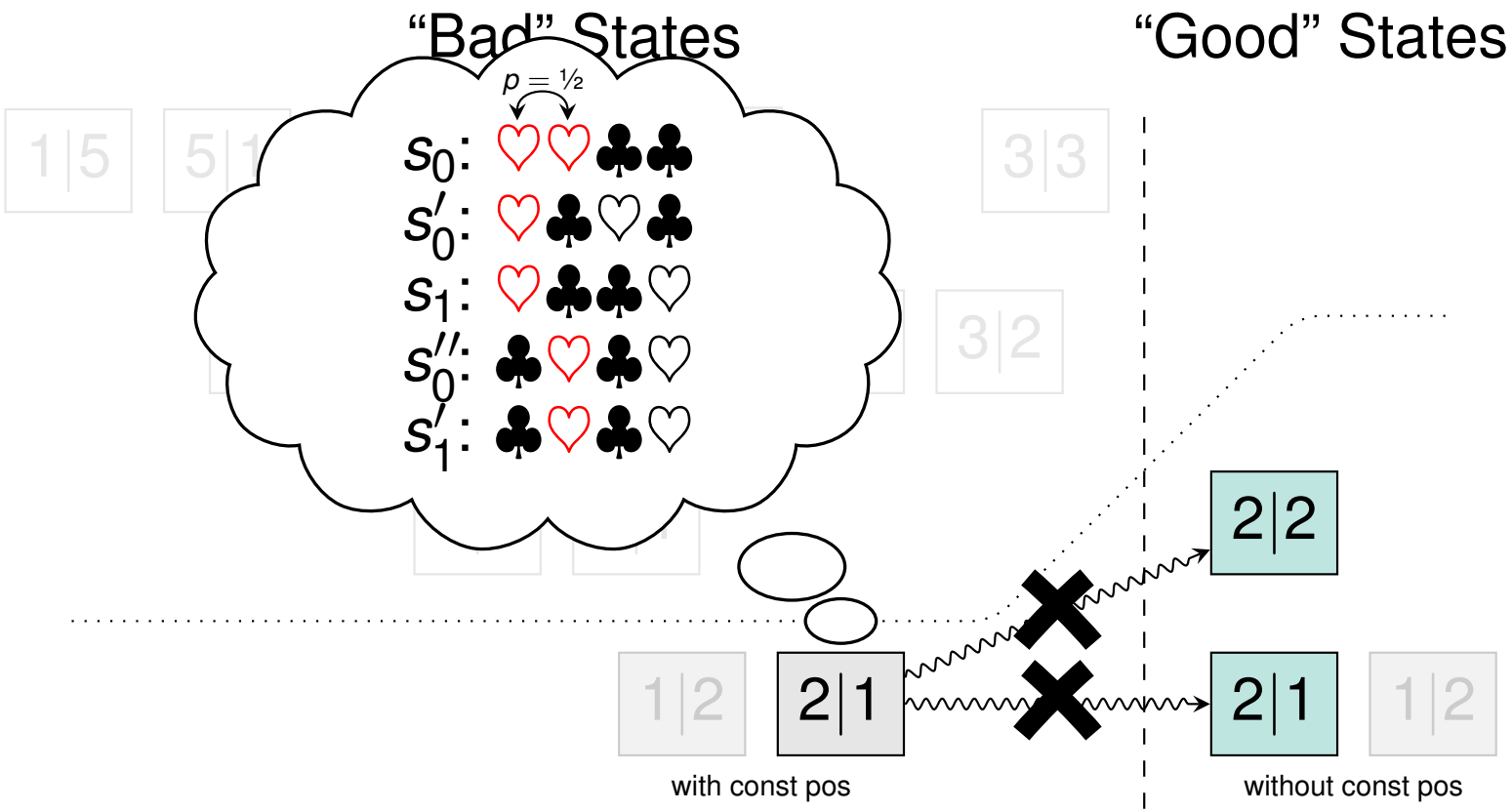
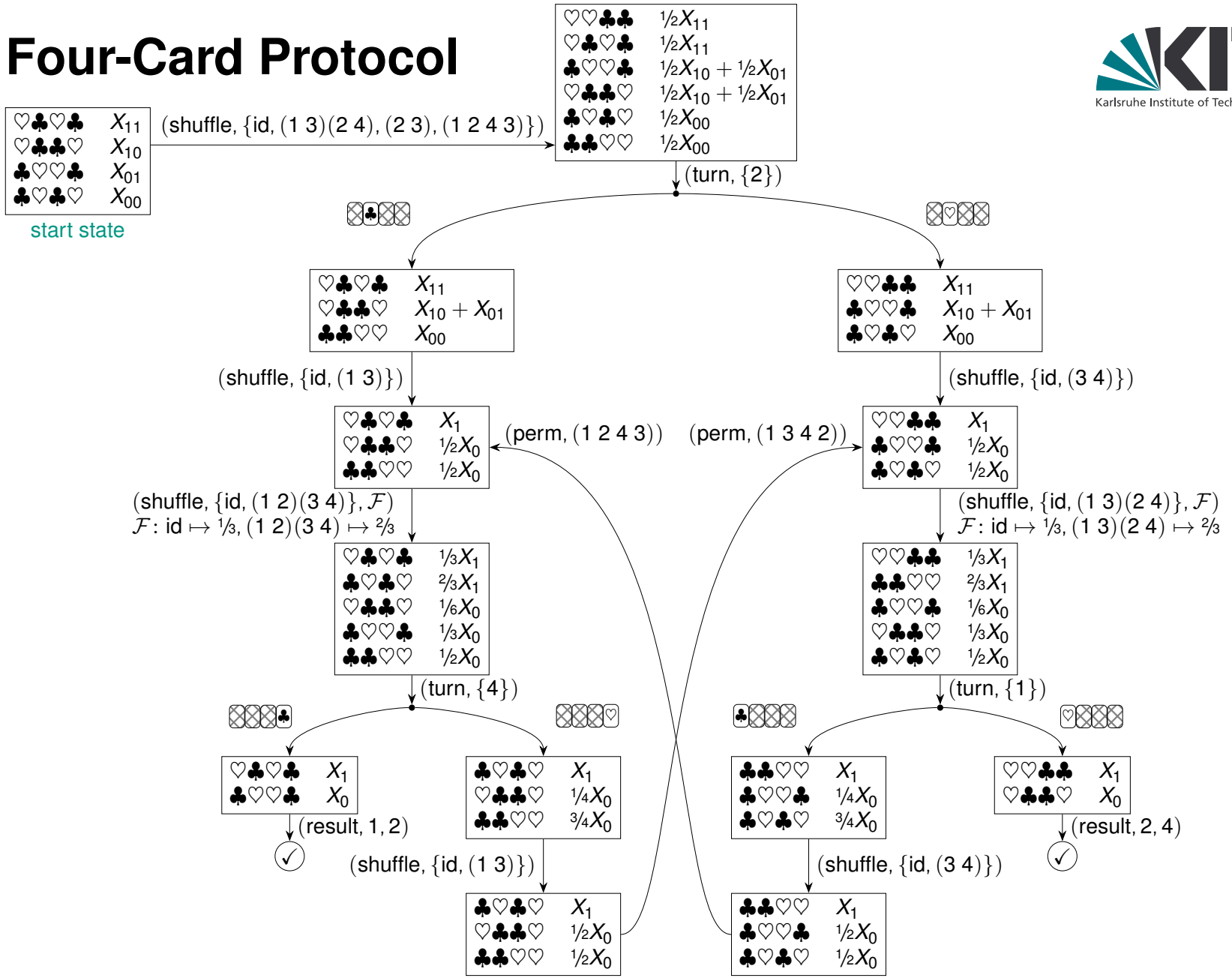


Apply (shuffle, Π, $\mathcal{F}$) to this state.

Case 2: There are $\pi_1, \pi_2 \in \Pi$ putting the const. col. in different pos.

$\Longrightarrow$ the resulting state has at least 5 sequences. □

Our Four-Card Protocol



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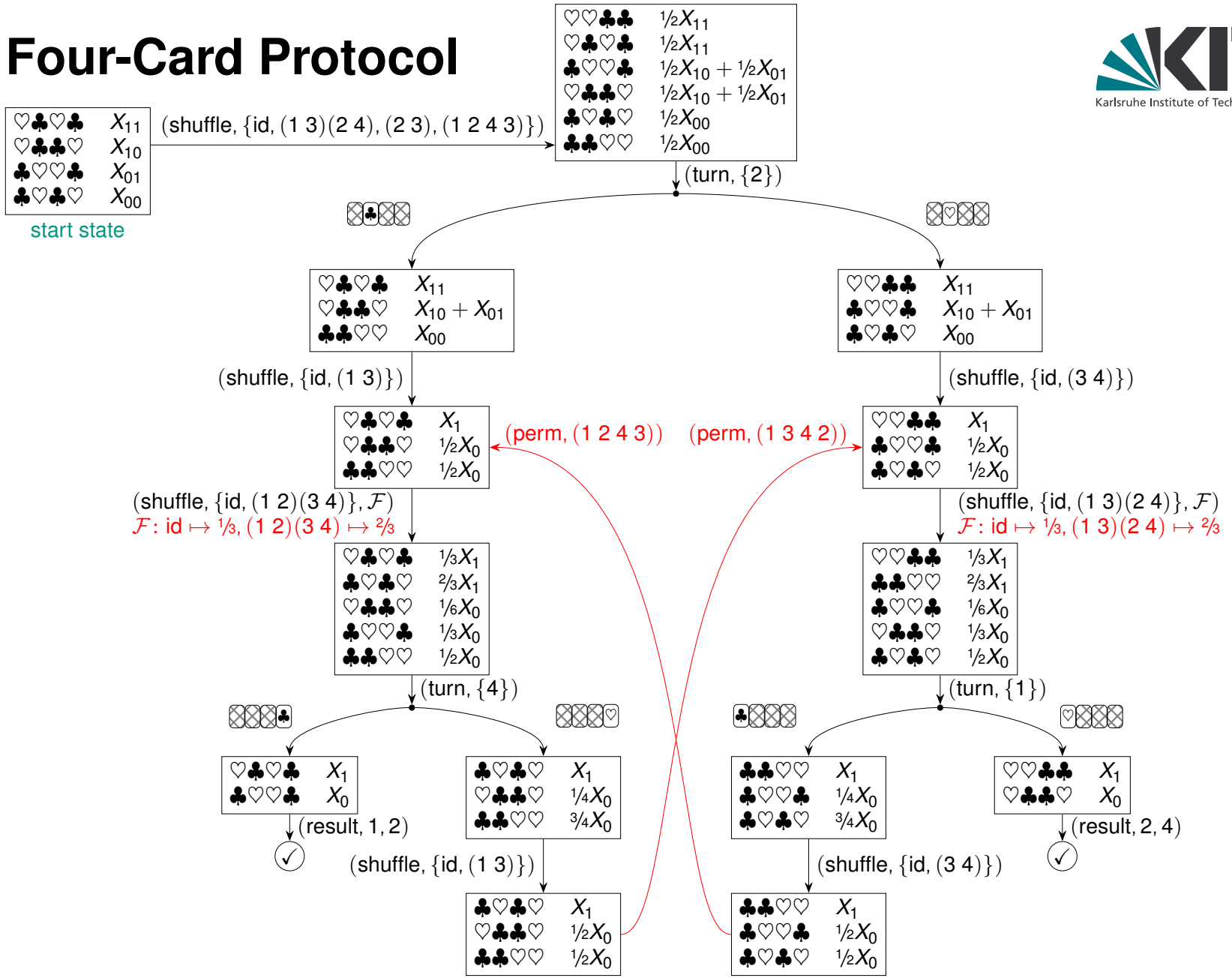
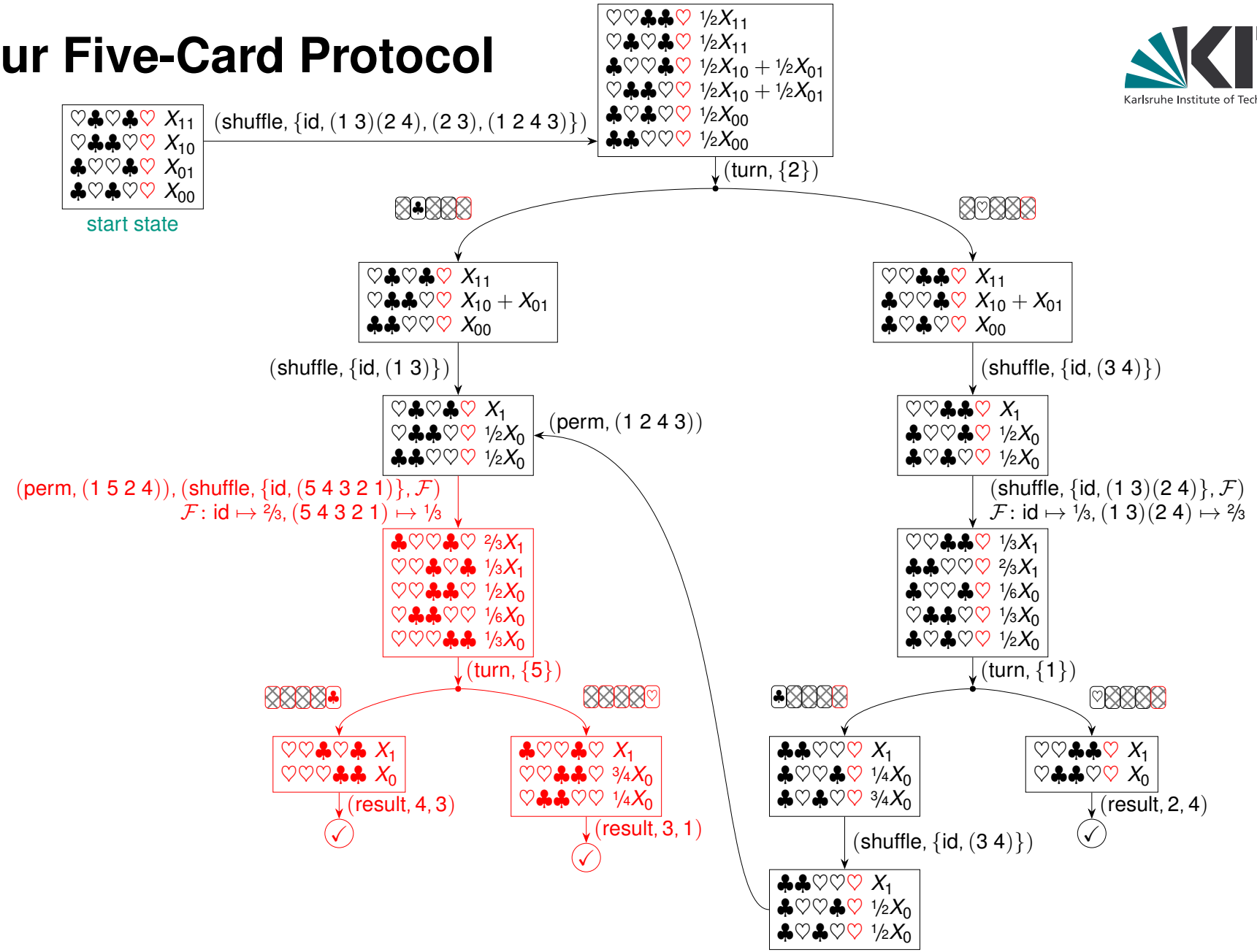
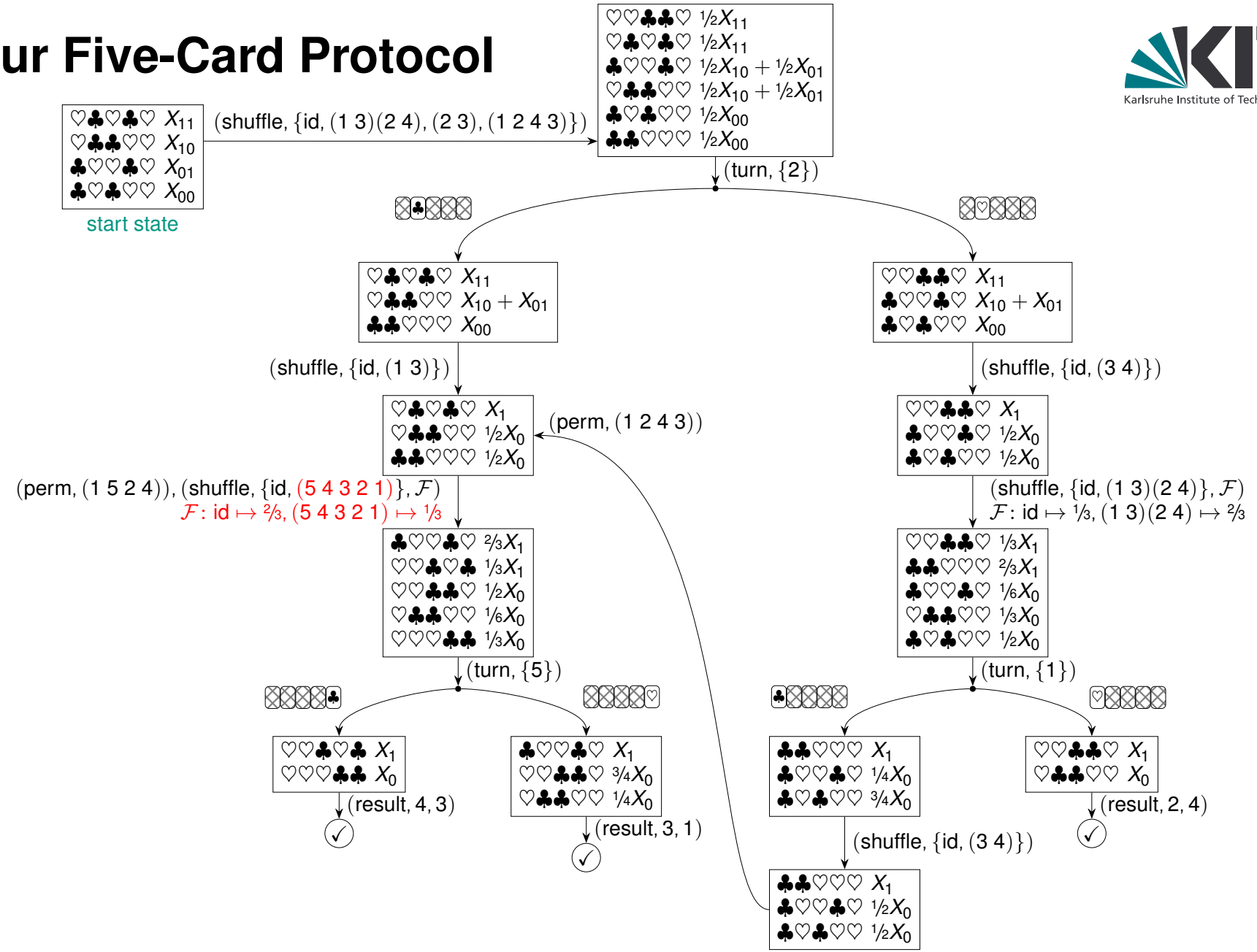


Our Five-Card Protocol



Our Five-Card Protocol



Summary

Runtime	Shuffles	#Cards	Reference
exp. finite	non-uniform closed	4	[KWH15]
exp. finite	uniform non-closed	4	[KWH15]
finite	non-uniform non-closed	5	[KWH15]
finite	uniform closed	≤ 6	[MS09]

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Open Question: What if we restrict the computational model?

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Thank you for your attention!

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Various Artists. Title image from http://pdpics.com/photo/6619-ten-cards-of-all-suits/, public domain. Image of Bar from https://pixabay.com/en/bar-pub-restaurant-rustic-barrels-406884/, public domain. Image of lecture hall from brett jordan, https://www.flickr.com/photos/x1brett/1472187414, CC-BY-2.0. XKCD comic figures by Randall Munroe from https://xkcd.com/, CC-BY-NC-2.5.