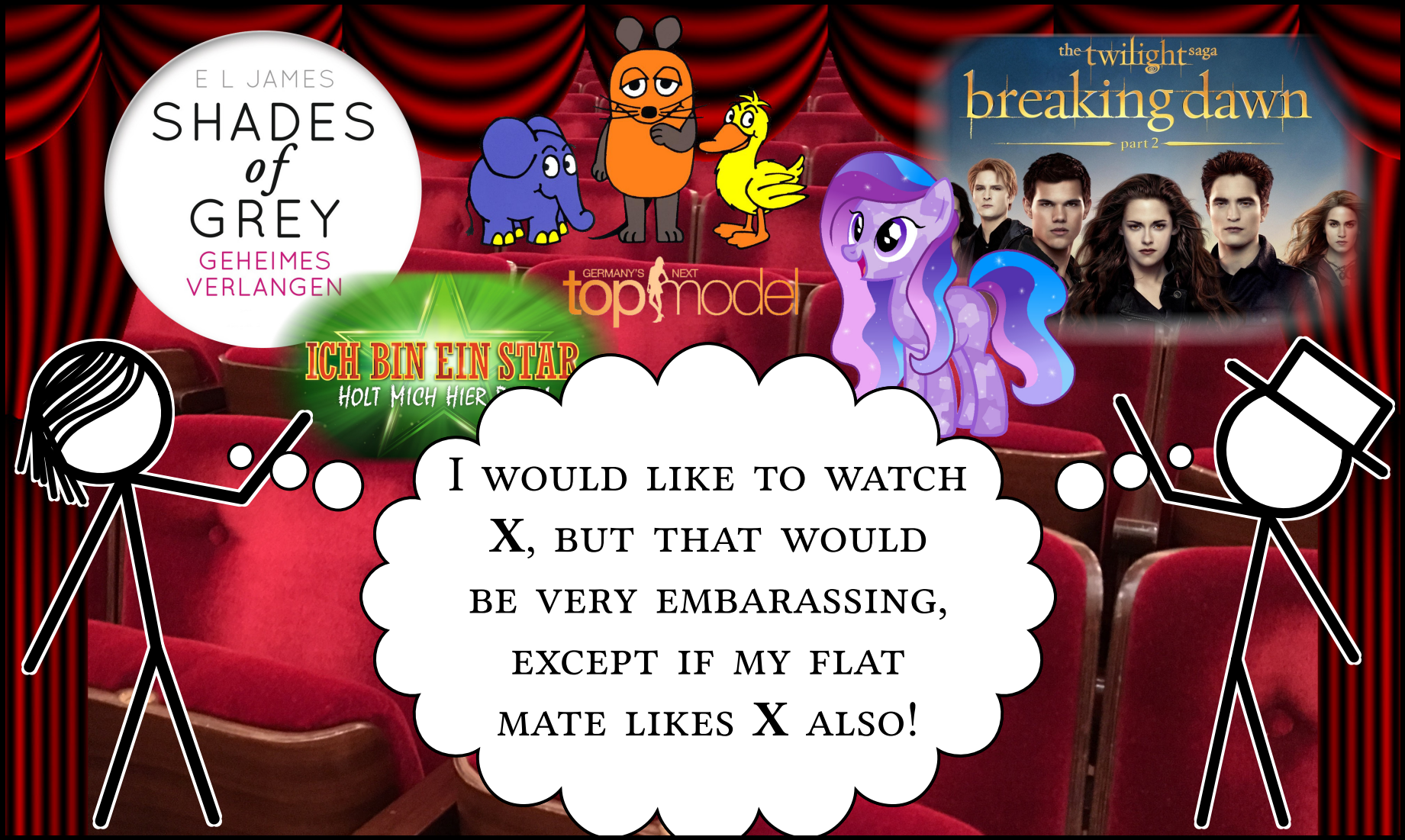
Karlsruhe Institute of Technology

The Minimum Number of Cards in Practical Card-based Protocols

ASIACRYPT 2017 | Julia Kastner, Alexander Koch, Stefan Walzer, Daiki Miyahara, Yu-ichi Hayashi, Takaaki Mizuki, Hideaki Sone

KARLSRUHE INSTITUTE OF TECHNOLOGY, TU ILMENAU, TŌHOKU UNIVERSITY, NARA INSTITUTE OF SCIENCE AND TECHNOLOGY

Problems at a Movie Evening...



Seats/curtains CC-0, Alice/Bob adapted from xkcd (by Randal Munroe) CC-BY-NC-2.5, logos copyrighted

First Try: Cryptographic Protocols in Software

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Motivation II: Didactic Contexts

Introduce cryptography to young people and students

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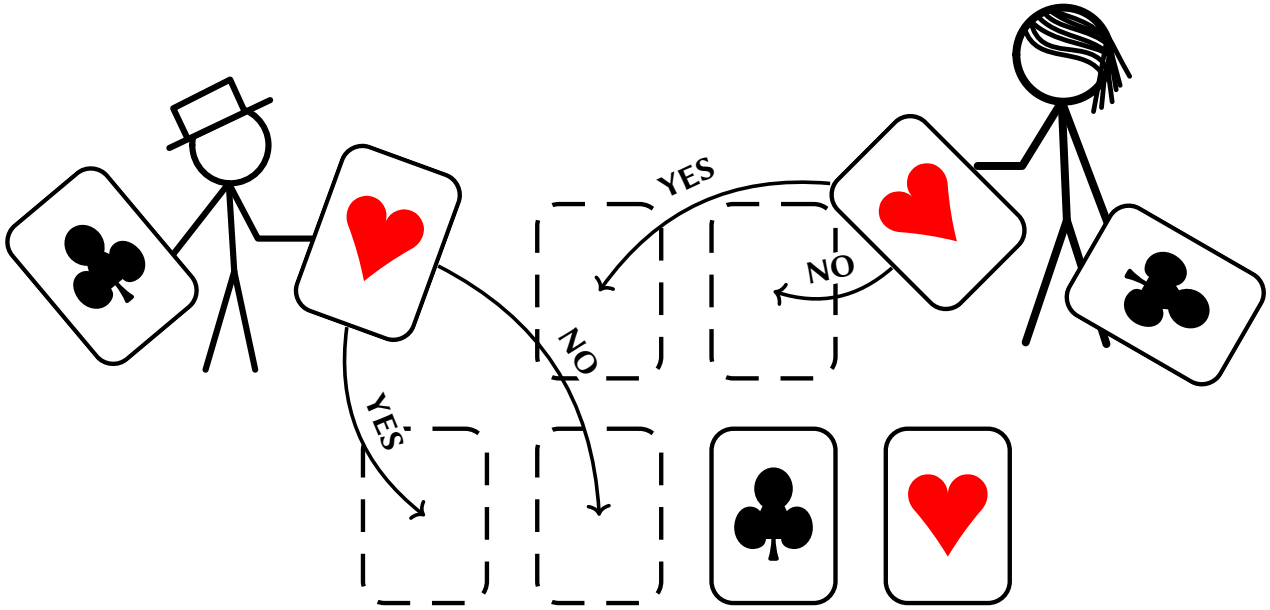
Computing AND with Six Cards [MS09]



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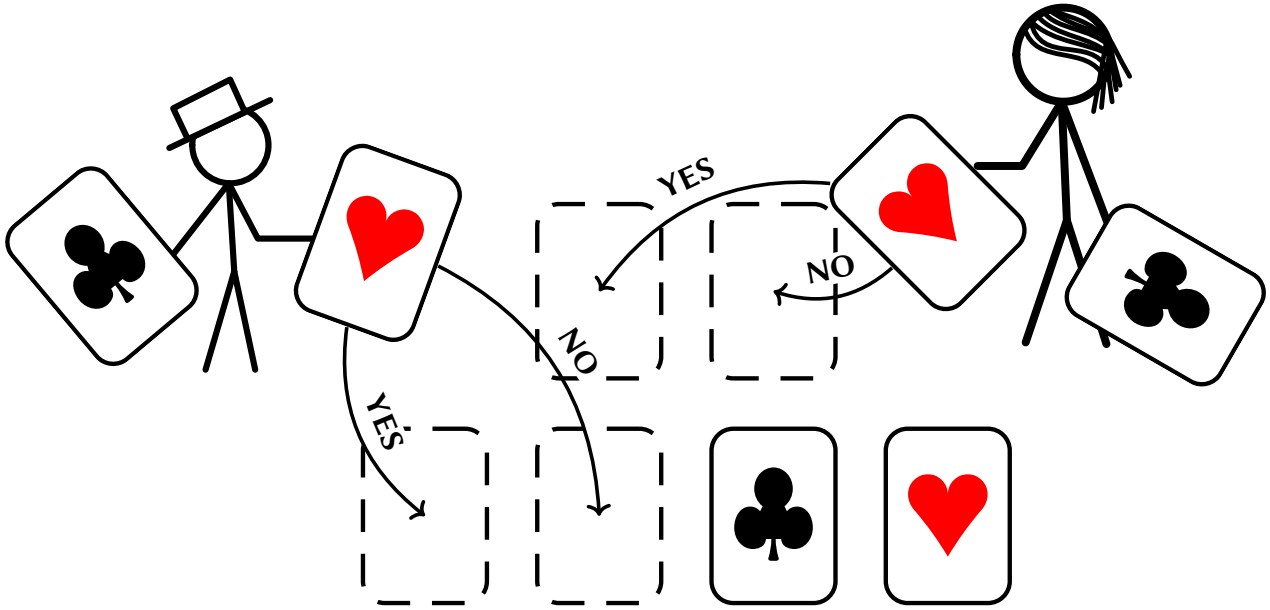
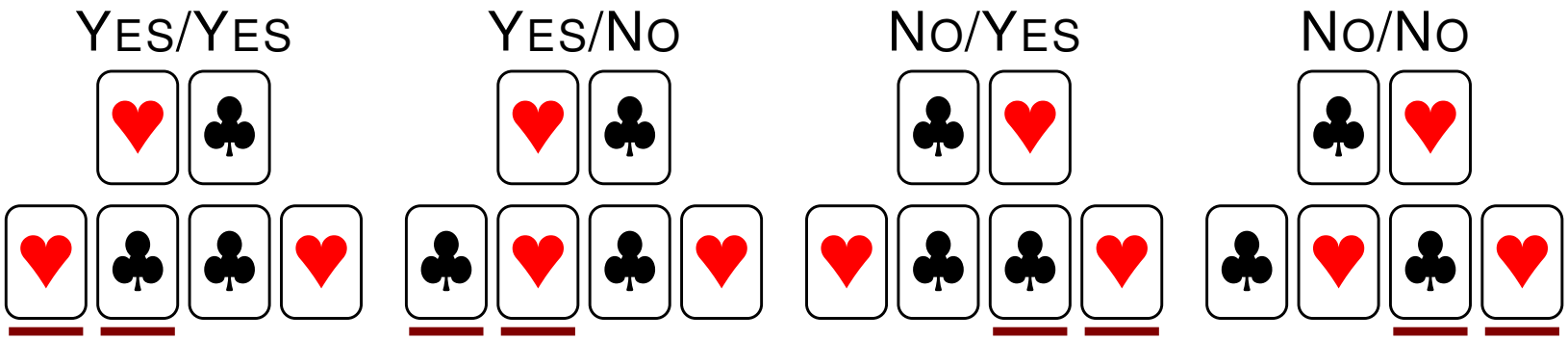


Configurations:



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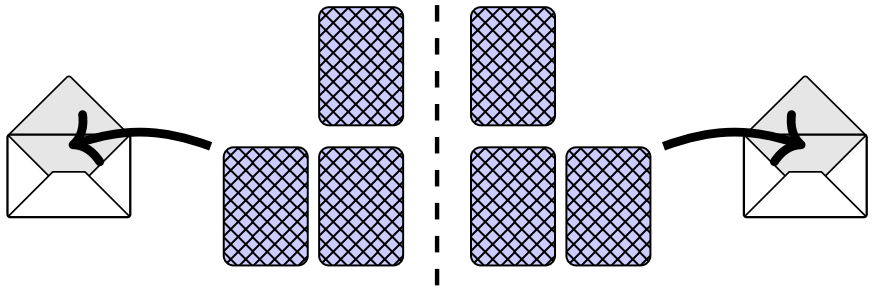

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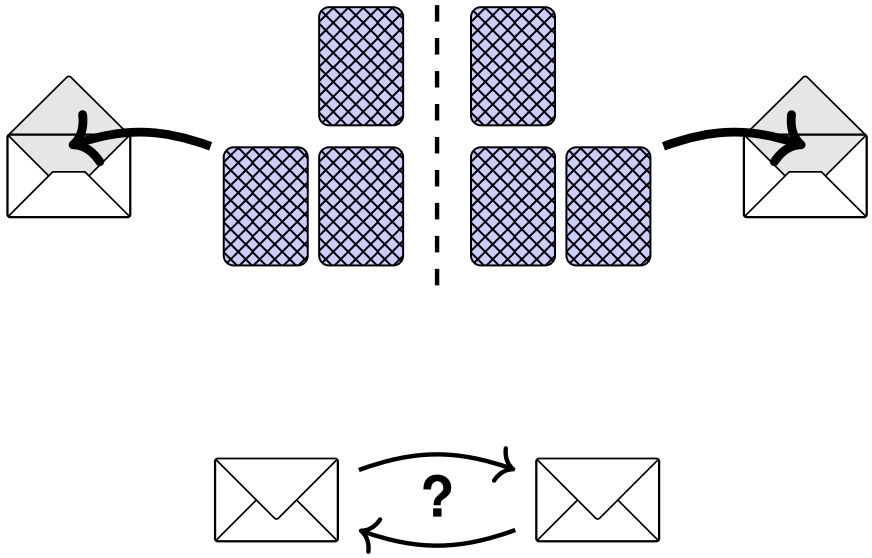

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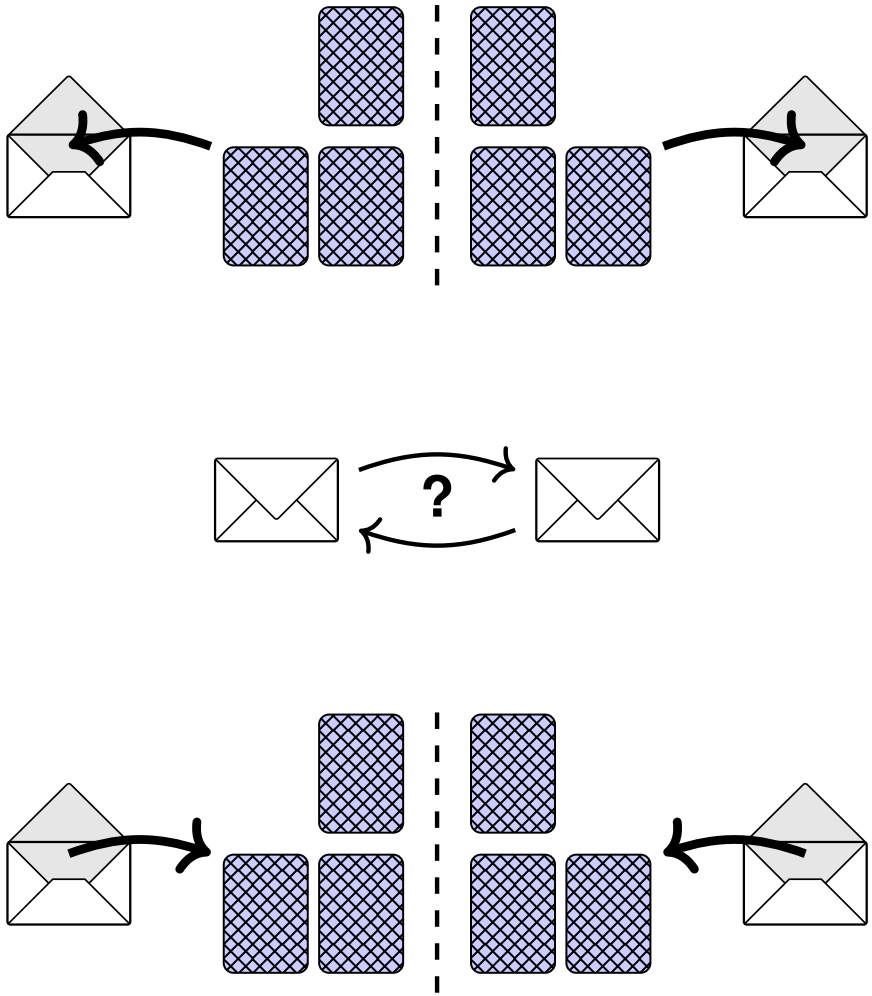

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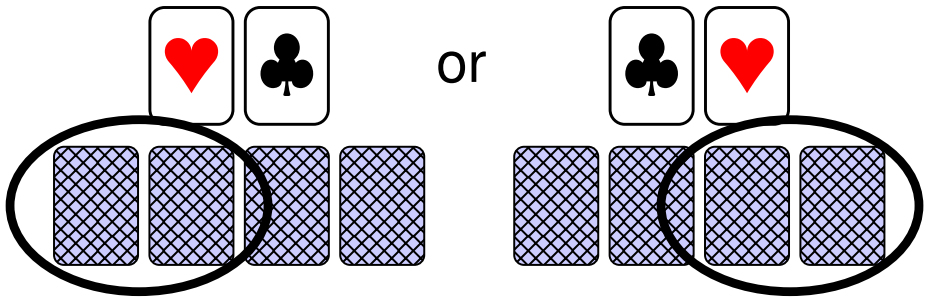

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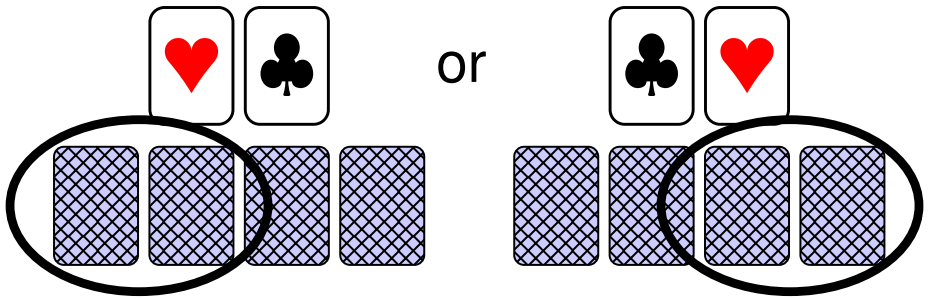
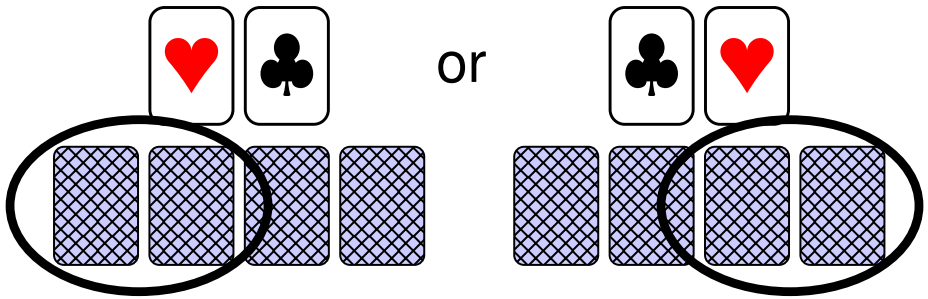


Main Question: 1. Can we do with less cards?

Computing AND with Six Cards [MS09]



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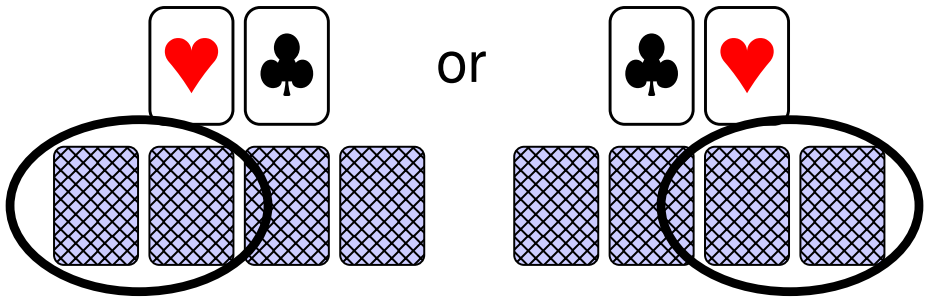

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And still be very practical?

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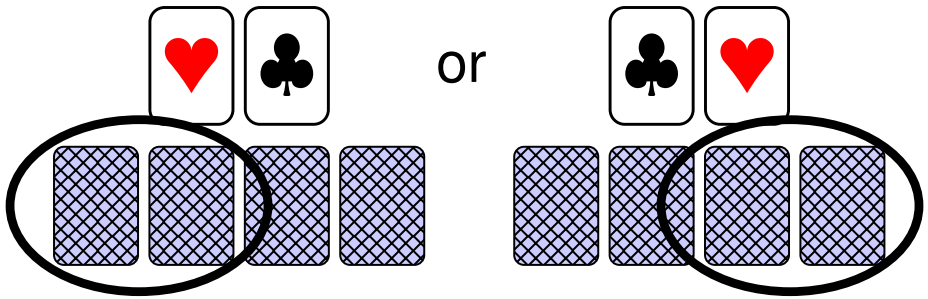

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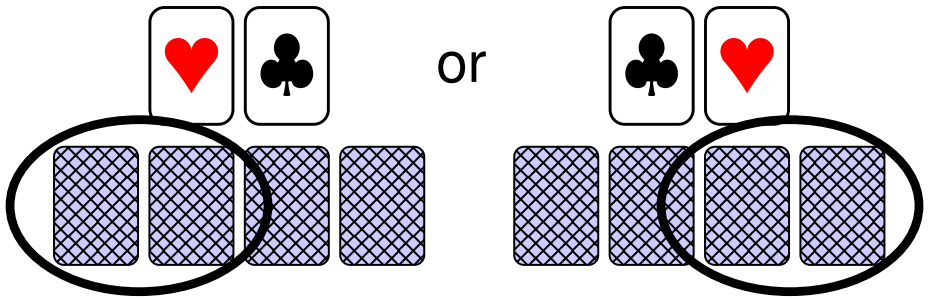


Main Question: 1. Can we do with less cards? Yes [KWH15]

And still be very practical? (Partially:) No (this talk).

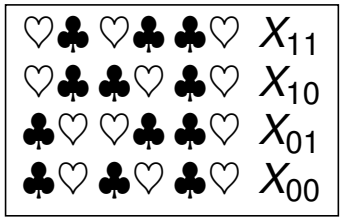
2. For arbitrary circuits, we additionally need COPY.
How many cards are necessary here?

State Tree of the Protocol

Karlsruher Institut für Technologie

Protocol State:

Currently possible sequences with symbolic input probability $X_{ij} = \Pr[\text{input} = (i, j)]$



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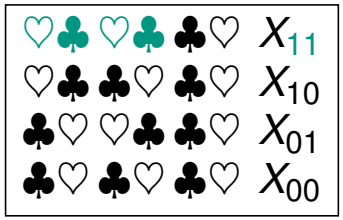

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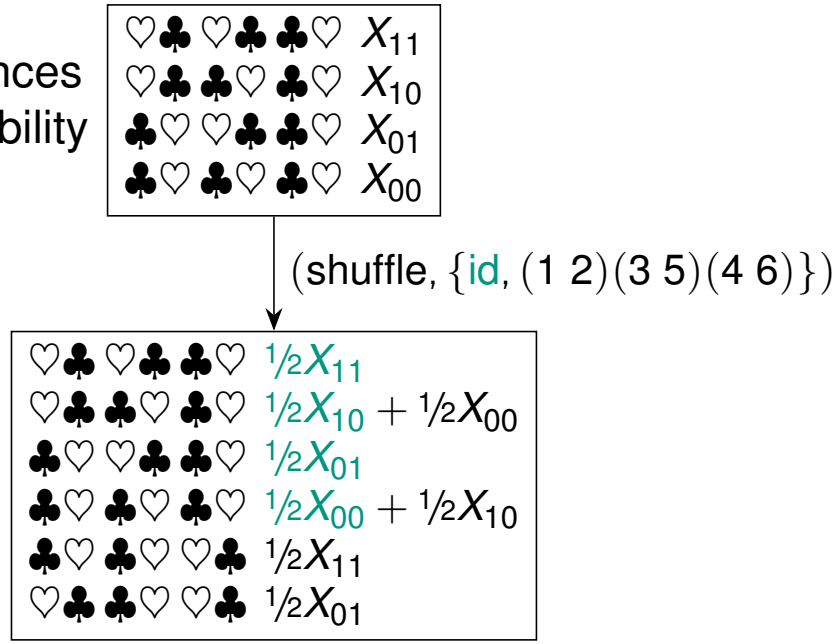

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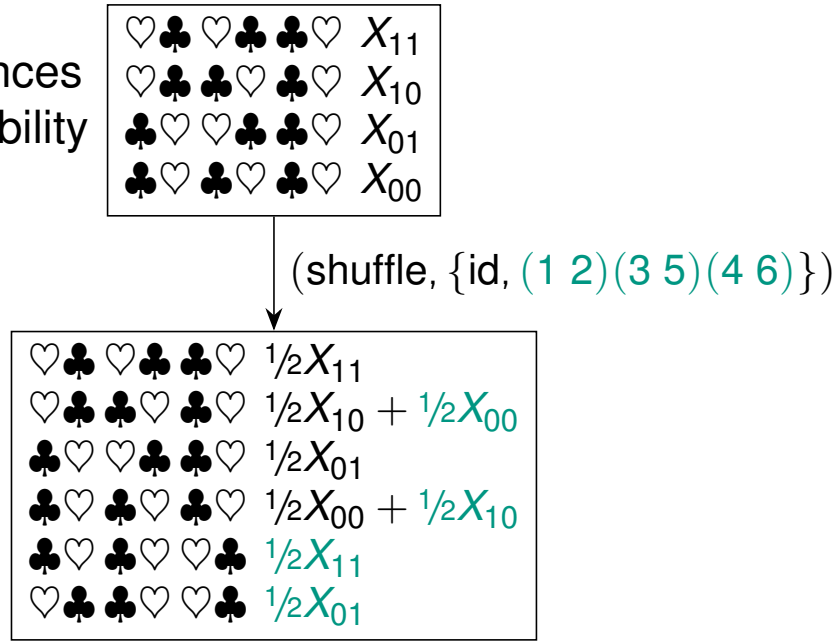

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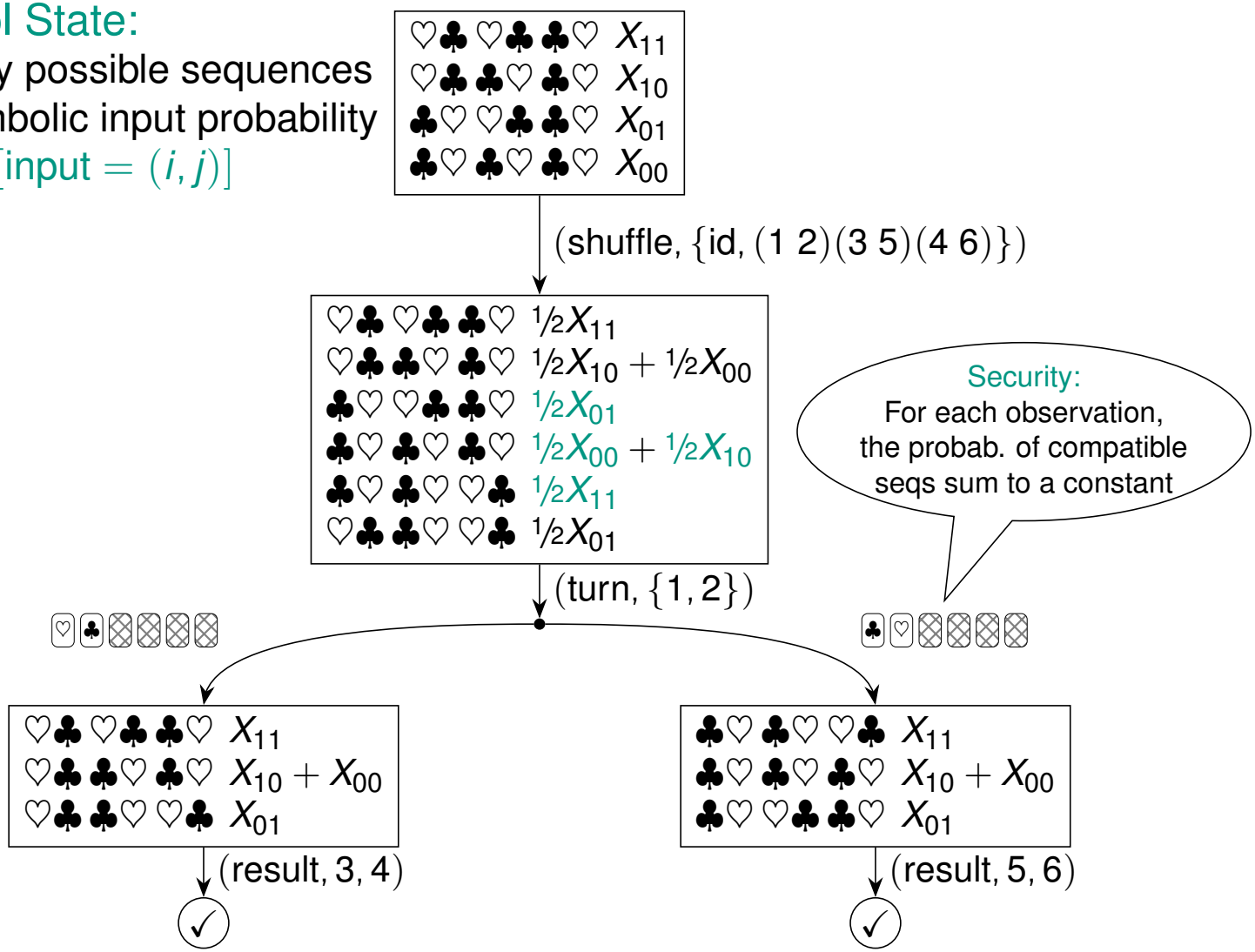

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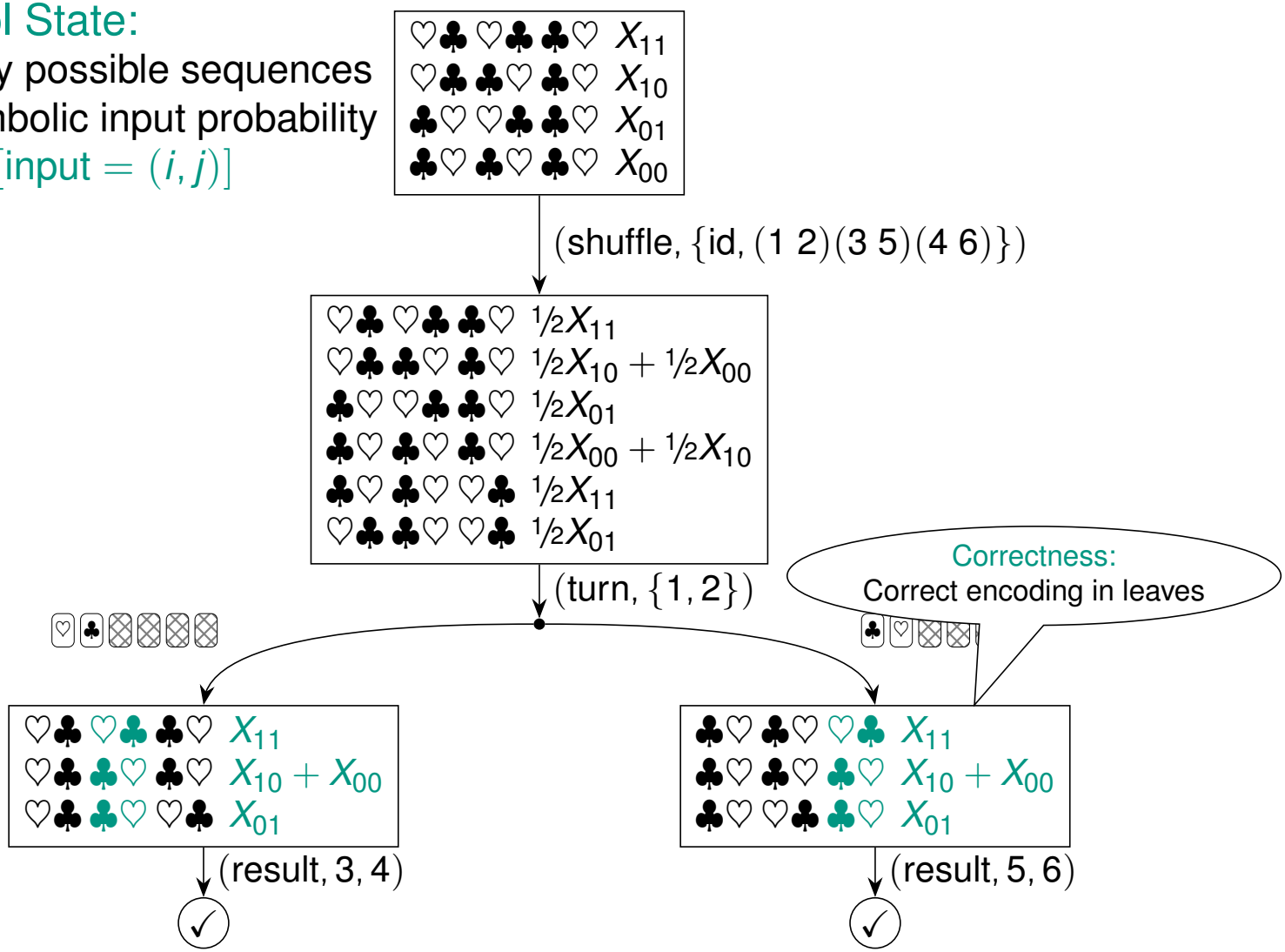
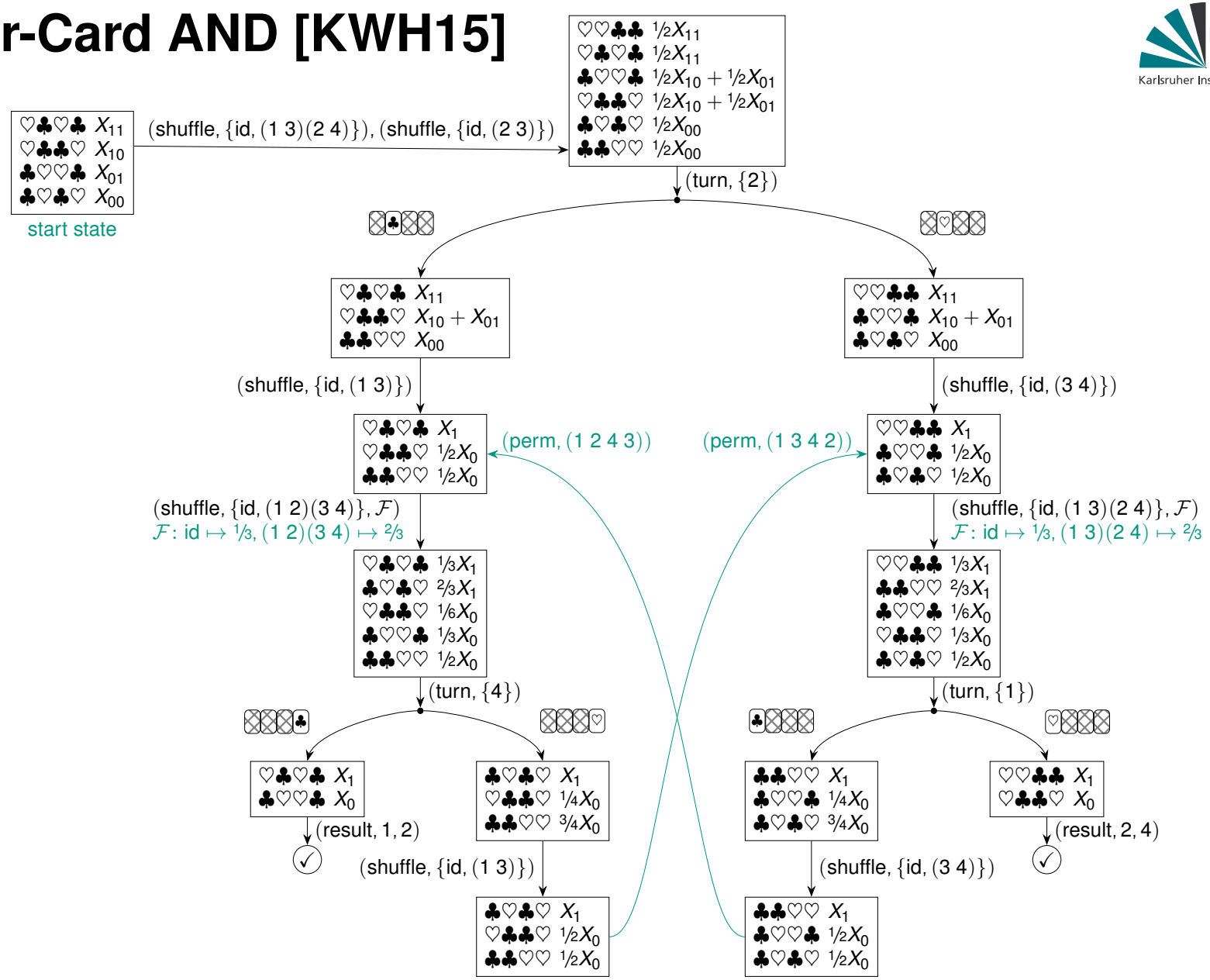
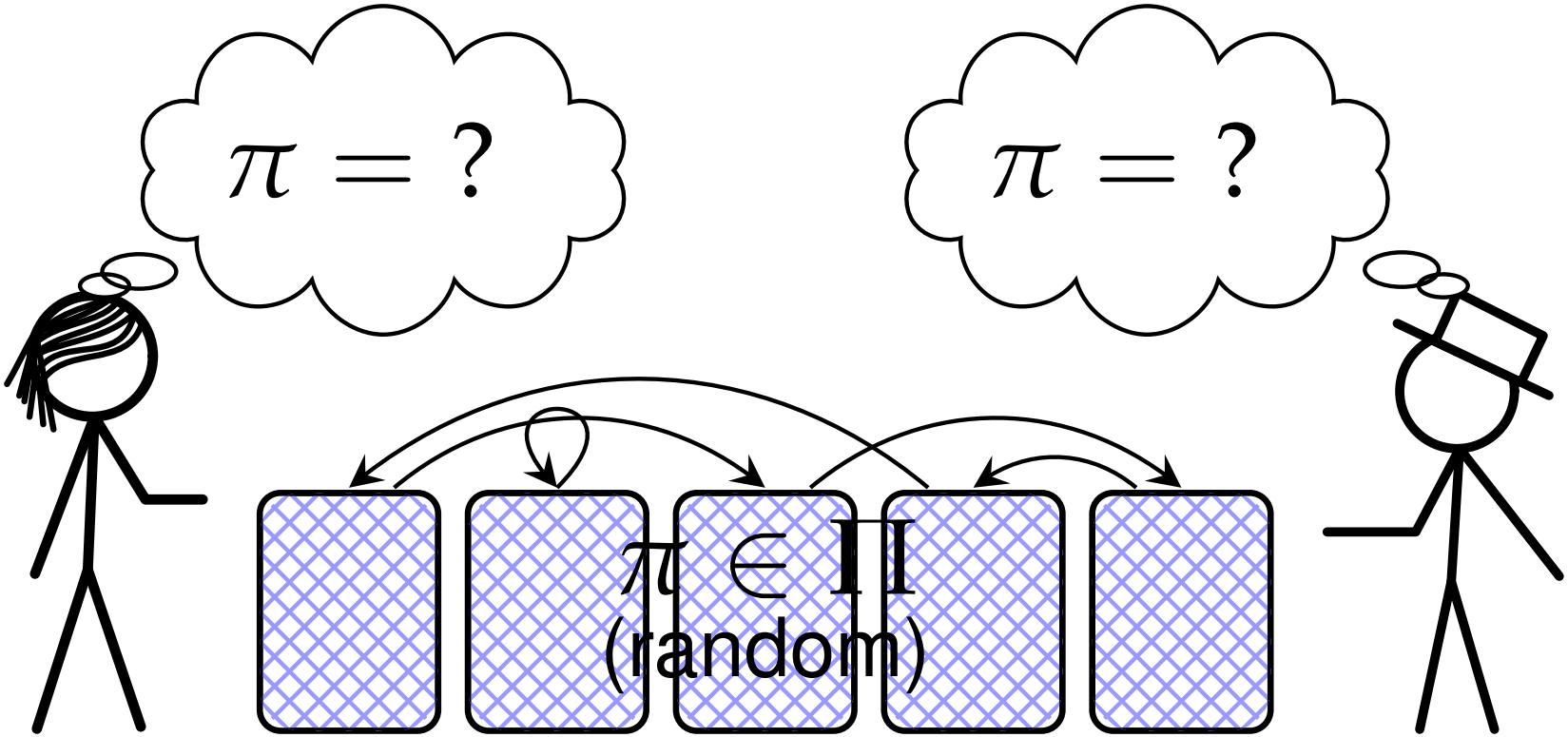


Four-Card AND [KWH15]



Why uniform closed shuffles are nice



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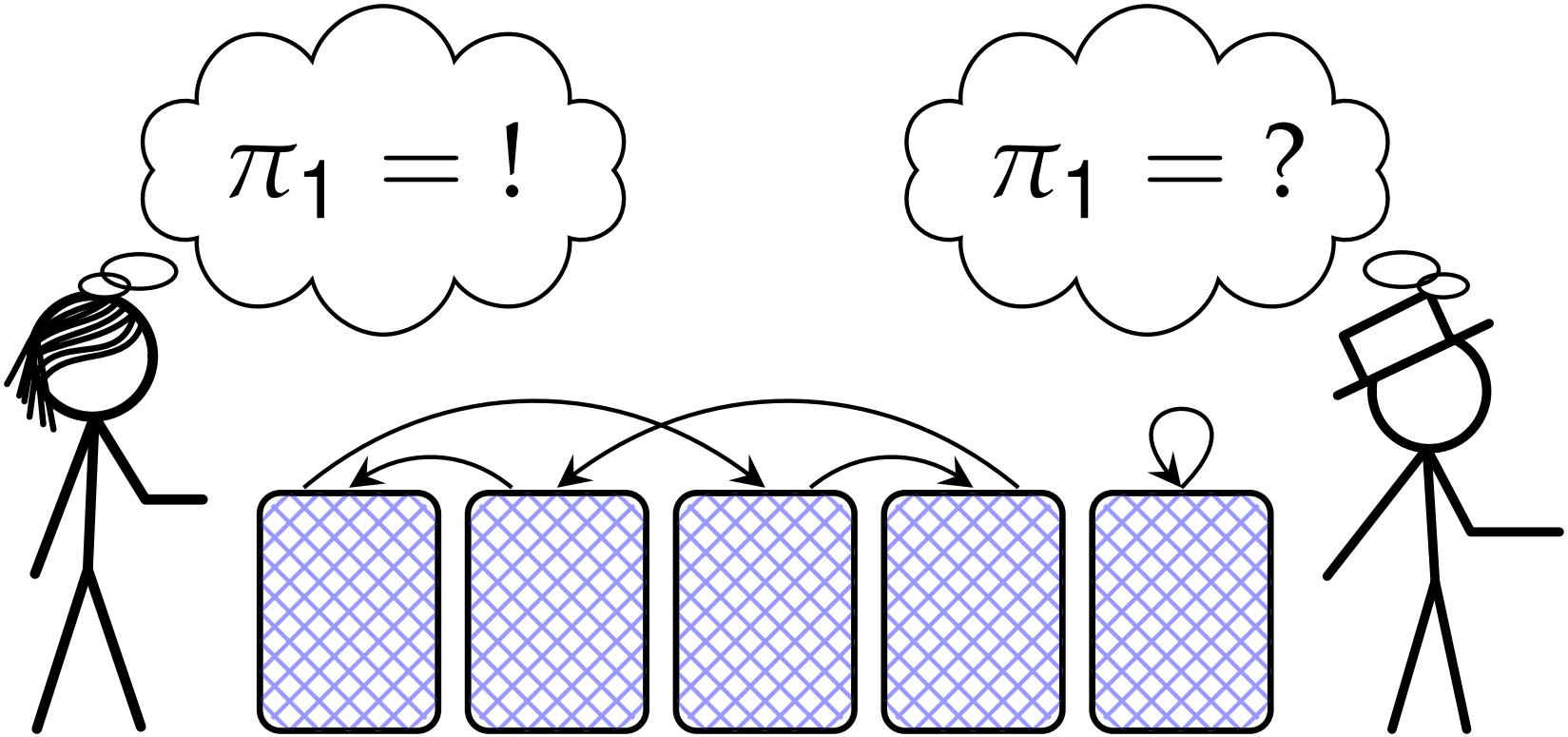

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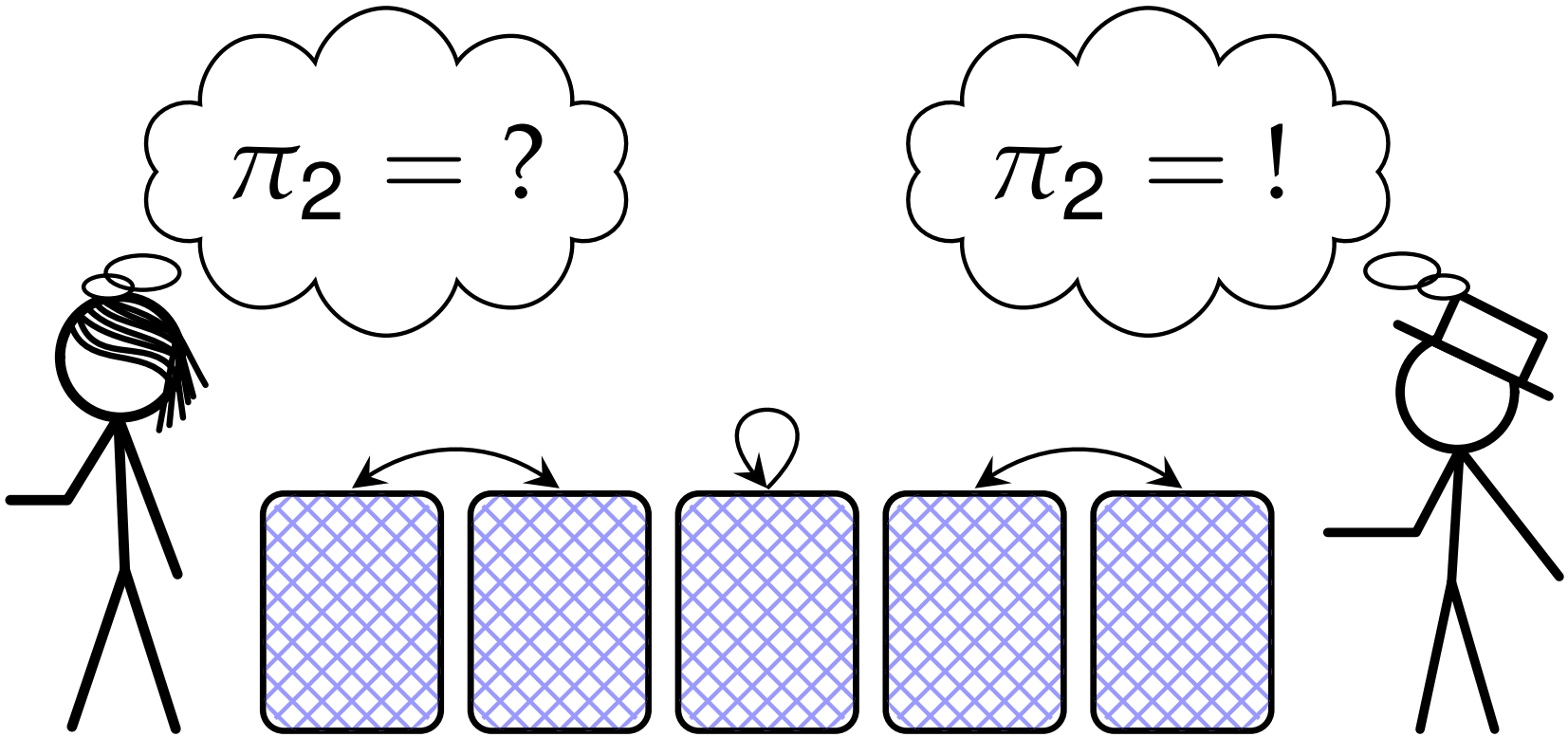

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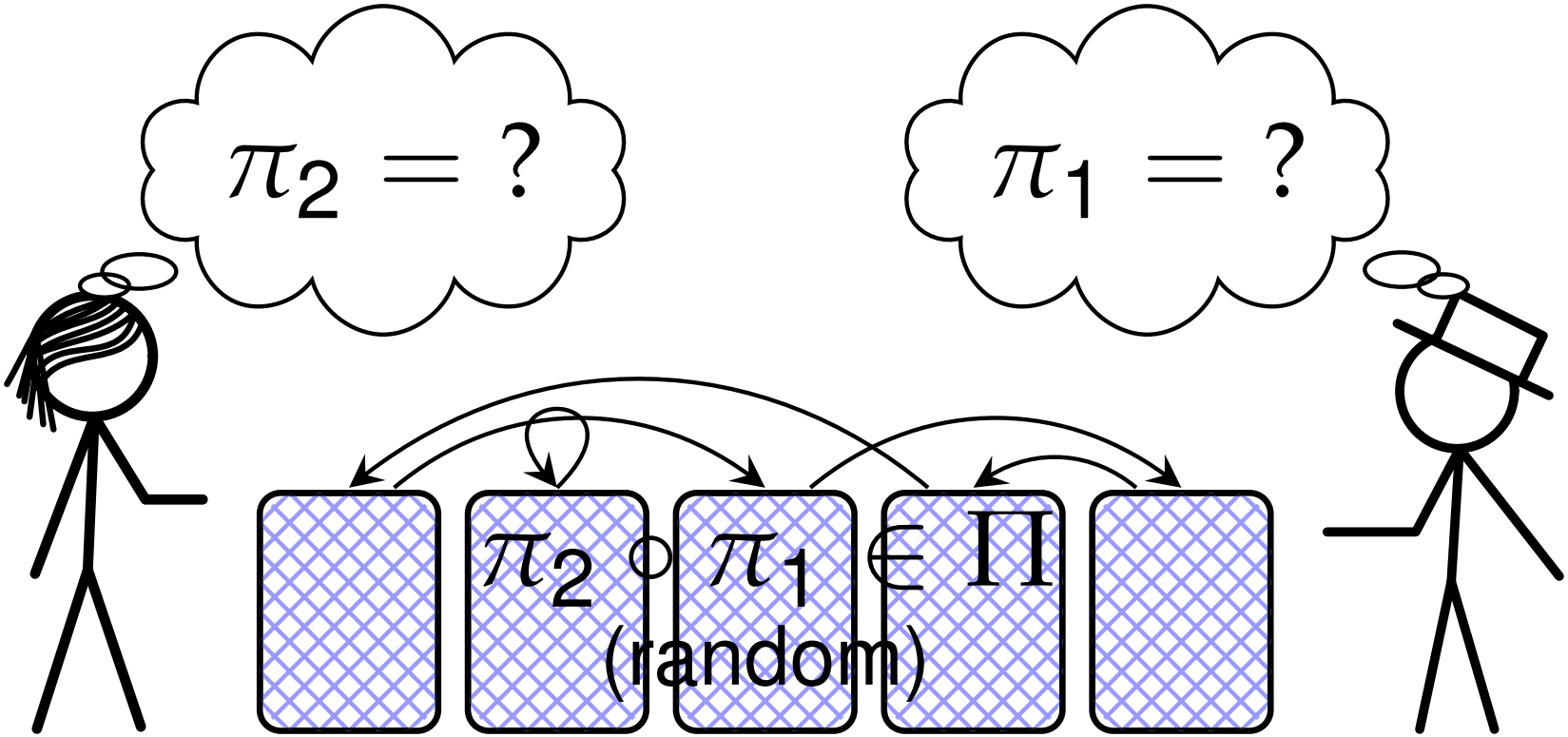

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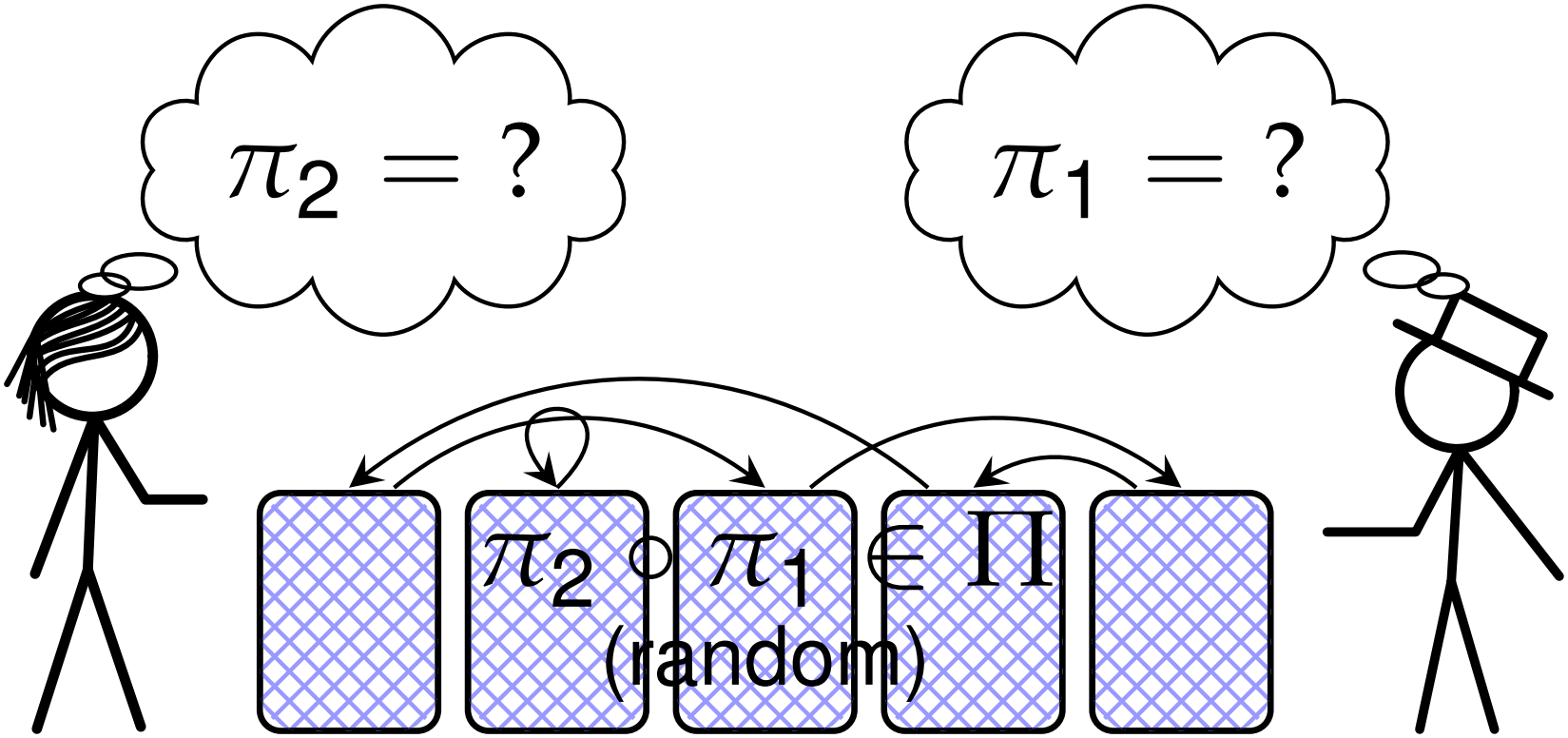
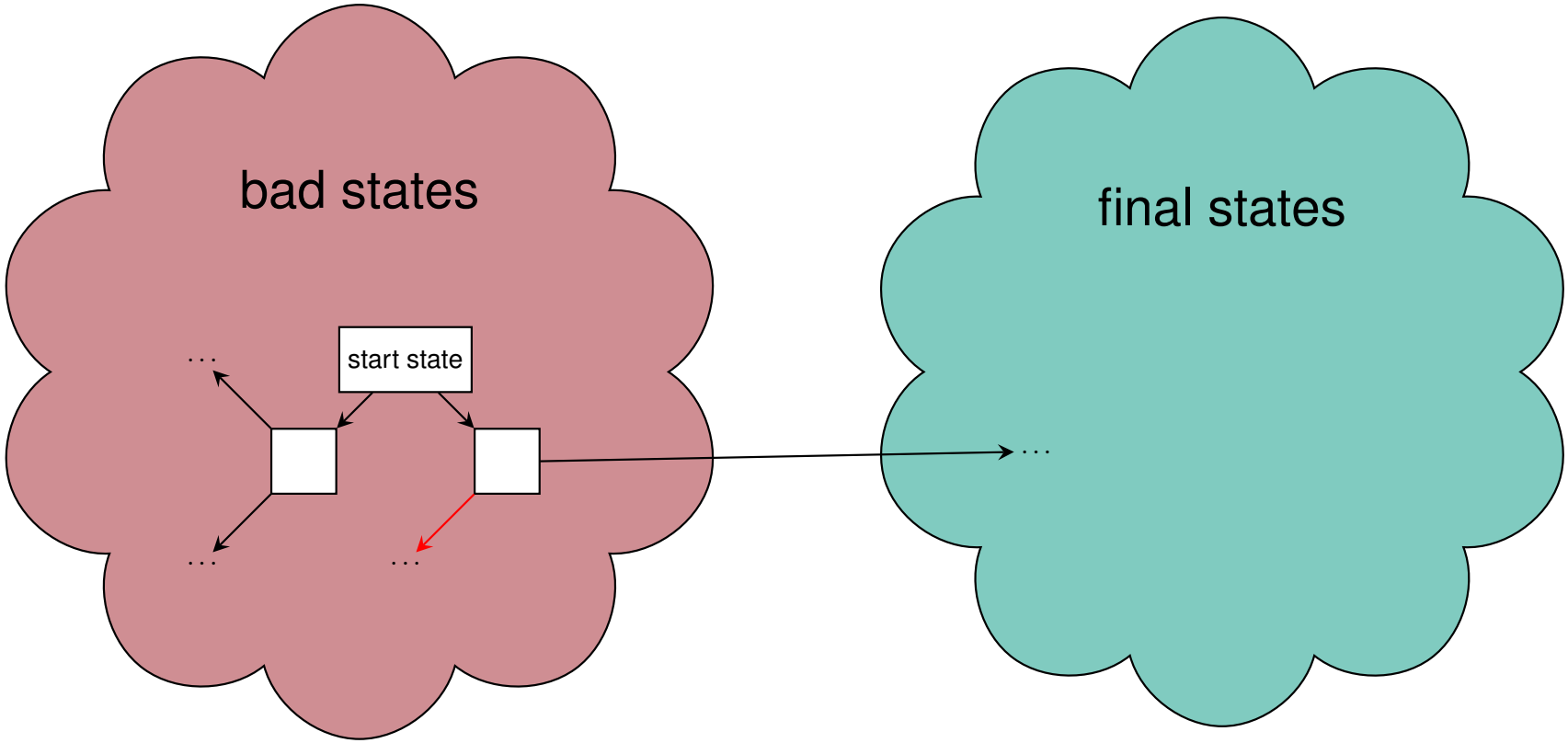


Note: there is an actively secure implementation using only uniform cuts and helping cards, where you do not look away [KW17].

Impossibility Result

Theorem
There is no secure finite-runtime closed-shuffle five-card AND protocol

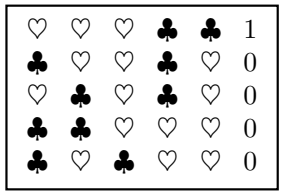
Proof Idea



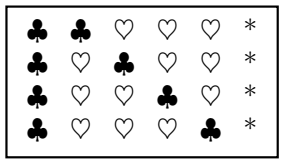
At least one outgoing edge (path) leads to a bad state again.

Proof Idea: Classification of States

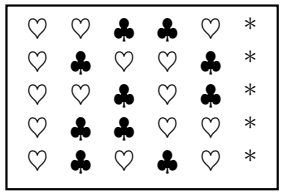
Bad States



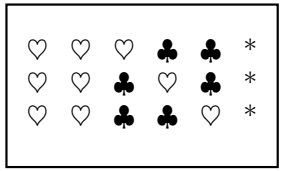
$\mathcal{B}_{4*}$
4 seqs of same type



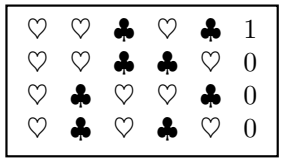
$\mathcal{B}_{3\clubsuit}$
♣-col, 3+ seqs



$\mathcal{B}_{5\heartsuit}$
♡-col, 5+ seqs

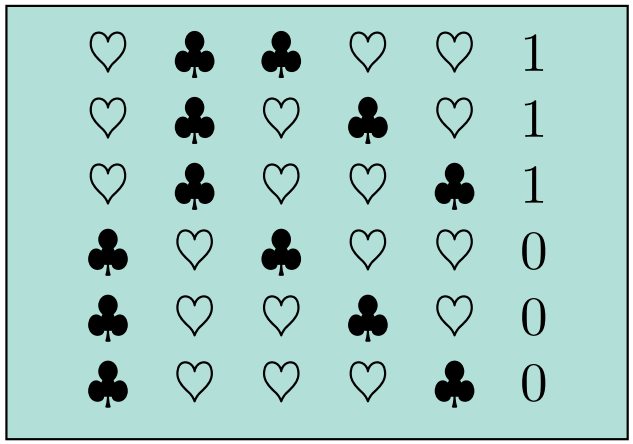


$\mathcal{B}_{3\heartsuit\heartsuit}$
2 ♡-cols, 3 seqs



$\mathcal{B}_{\heartsuit 3/1}$
♡-col, type 3/1 or 1/3

Final States

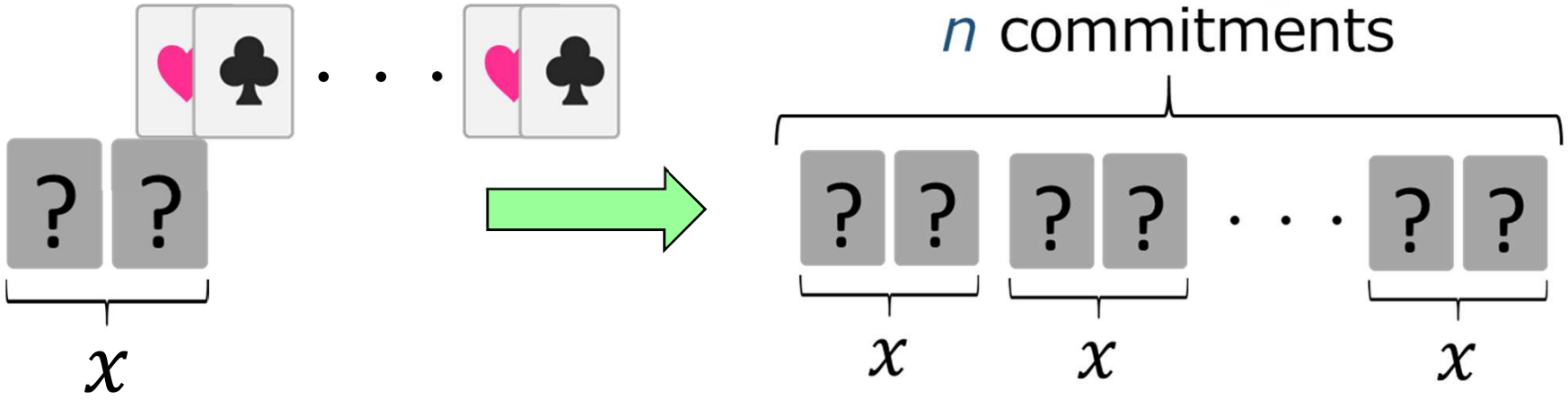


and any subset with at least a 1- and a 0-seq

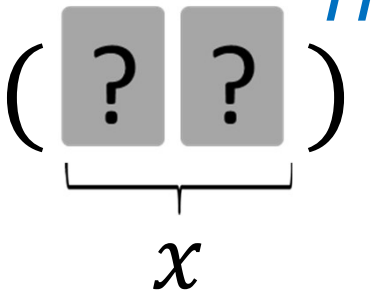
COPY protocols

✓Making n (≥ 2) copied commitments from an input commitment.

At least $2n$ cards are necessary.



✓We sometimes write $(??)^n$



The state-of-the-art COPY protocols

	# cards	Runtime
Mizuki-Sone [FAW09]	$2n+2$	Finite
Nishimura et al. [Soft Com.17]	$2n+1$	Las Vegas

Contribution

✓We show lower bounds on the numbers of cards:

- ✓$2n+1$ cards are required for any COPY protocol;
- ✓$2n+2$ cards are necessary for finite-runtime.

These are optimal in terms of the number of required cards

	# cards	Runtime
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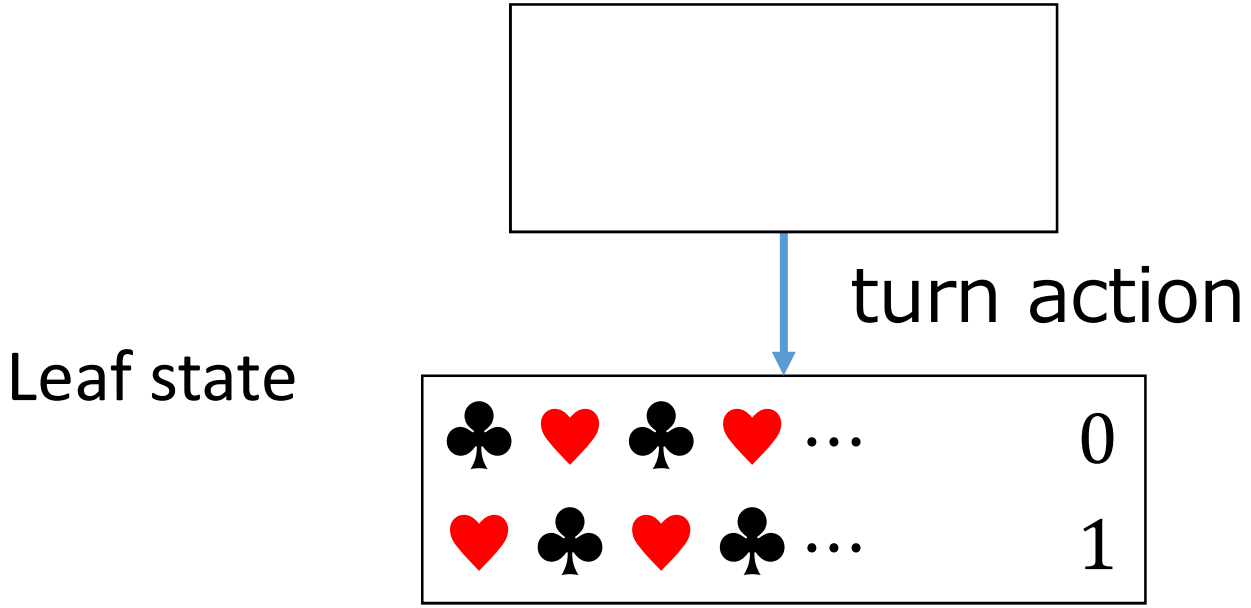
Impossibility with $2n$ cards

- ✓The proof outline:
  - ✓Assume the <u>existence</u> of COPY protocols with $2n$ cards,



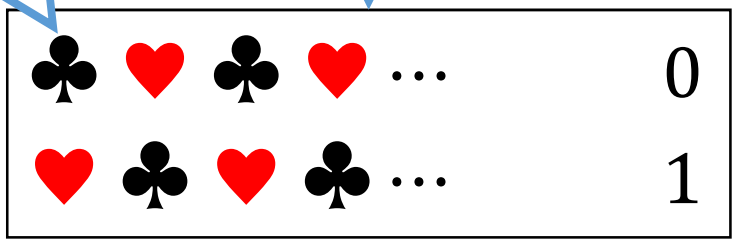
Impossibility with $2n$ cards

- ✓The proof outline:
  - ✓Assume the existence of COPY protocols with $2n$ cards,

Both the turned cards must be the same color, a contradiction.

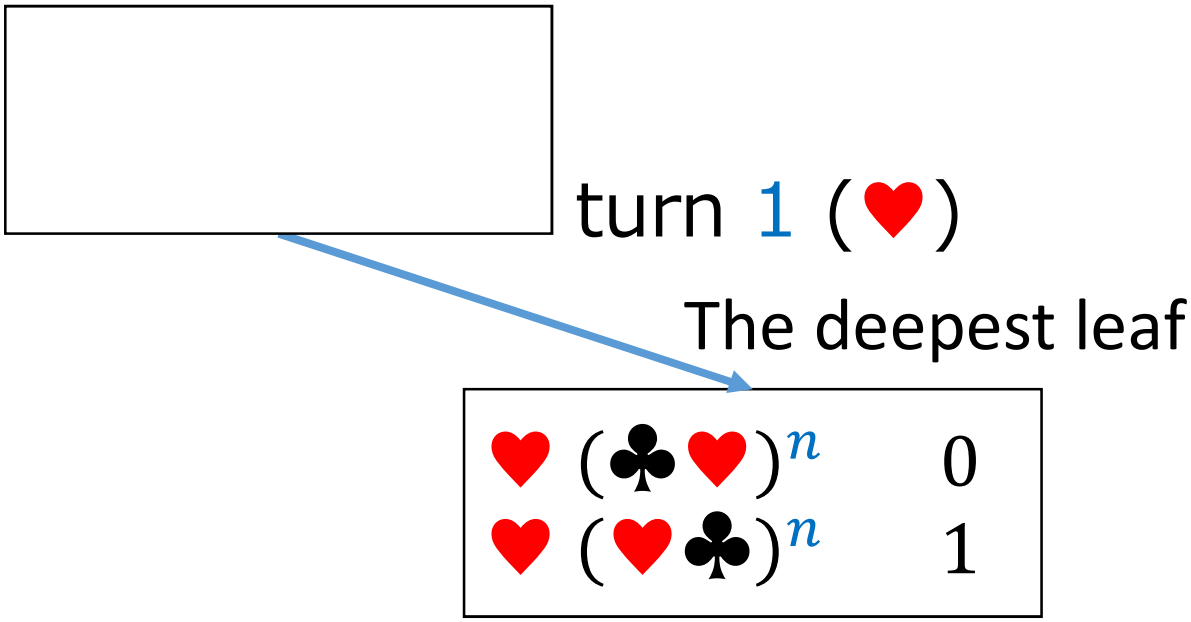
turn action

Leaf state



Impossibility with $2n+1$ cards for finite

- ✓The proof outline:
  - ✓Assume the existence of finite COPY with $\clubsuit^n, \heartsuit^{n+1}$.
  - ✓There must be the deepest leaf.



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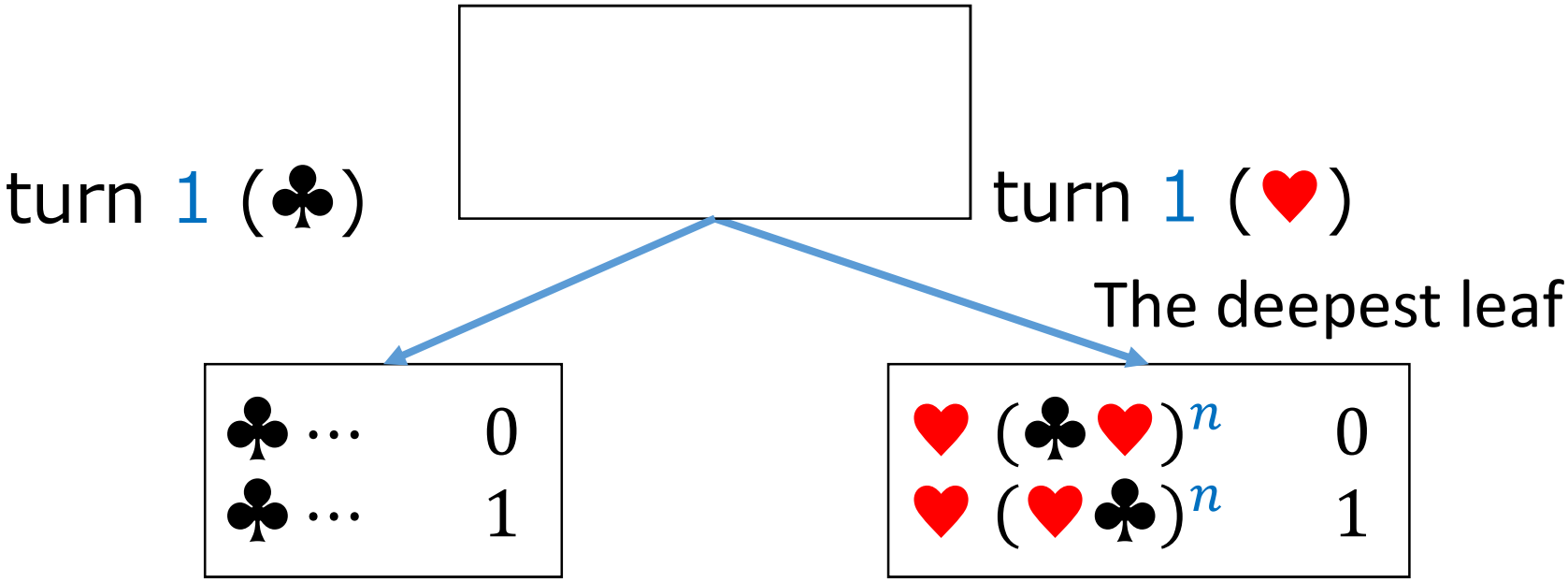

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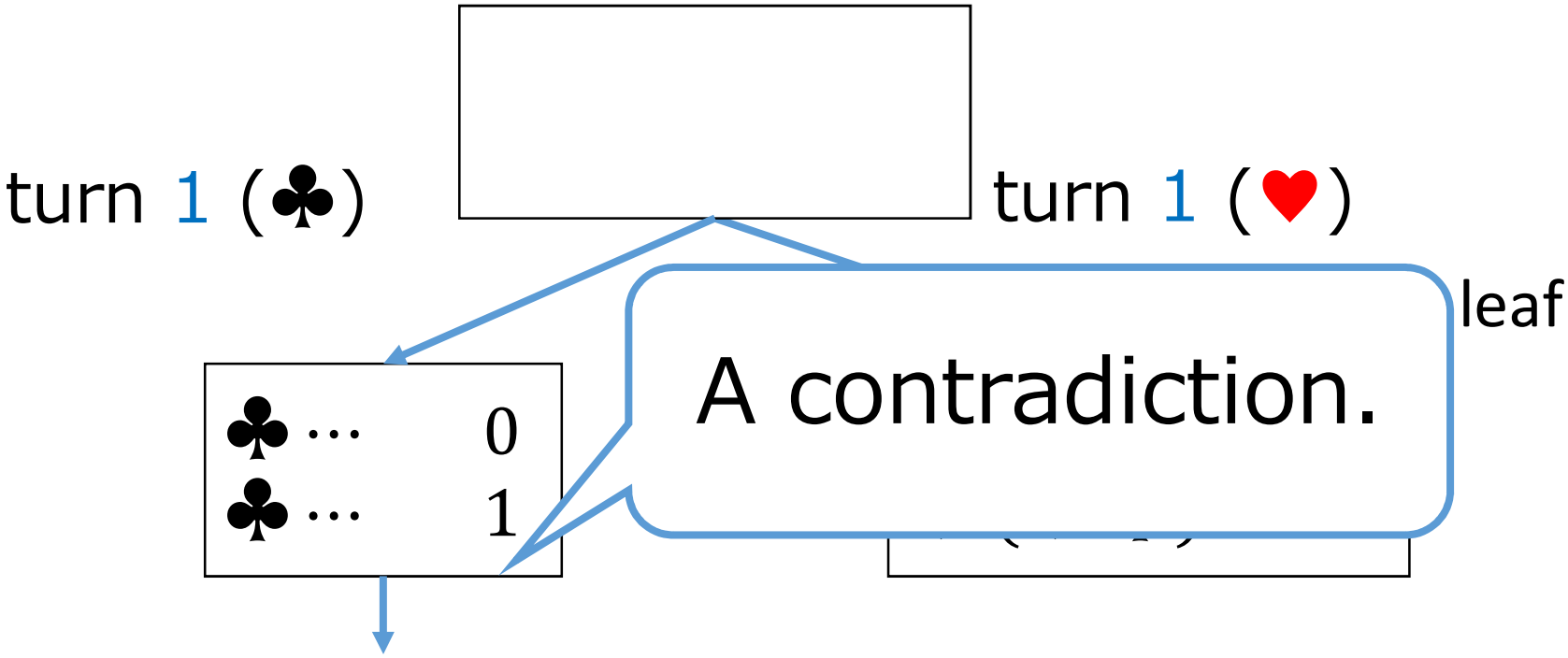
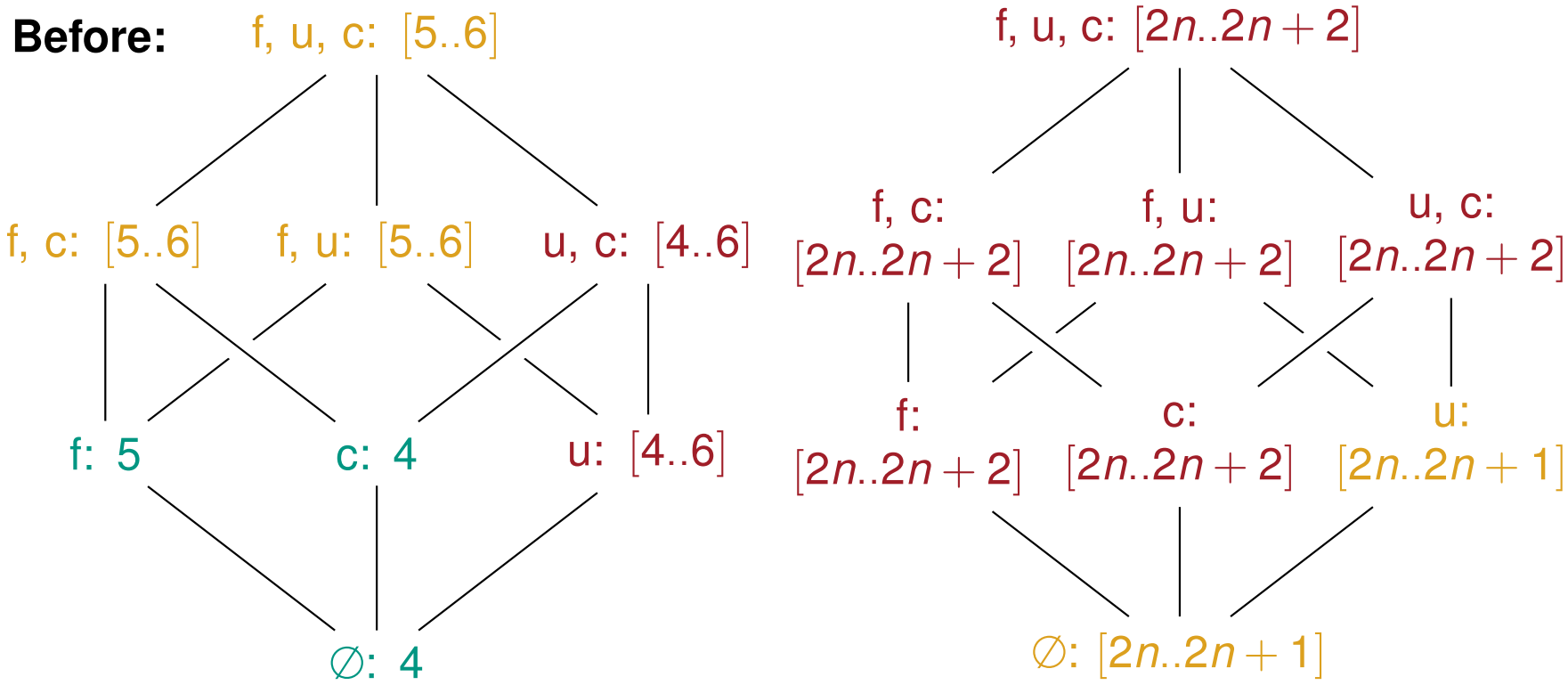


- ✓Because we cannot construct n commitments with $\clubsuit^{n-1}$ and $\heartsuit^{n+1}$, there should be a deeper leaf.

Summary

- 6 cards needed for finite-runtime **(f)** AND with closed **(c)** shuffles
- 5 cards needed for AND with uniform **(u)** closed shuffles
- $2n+1$ cards needed for COPY
- $2n+2$ cards needed for finite-runtime COPY



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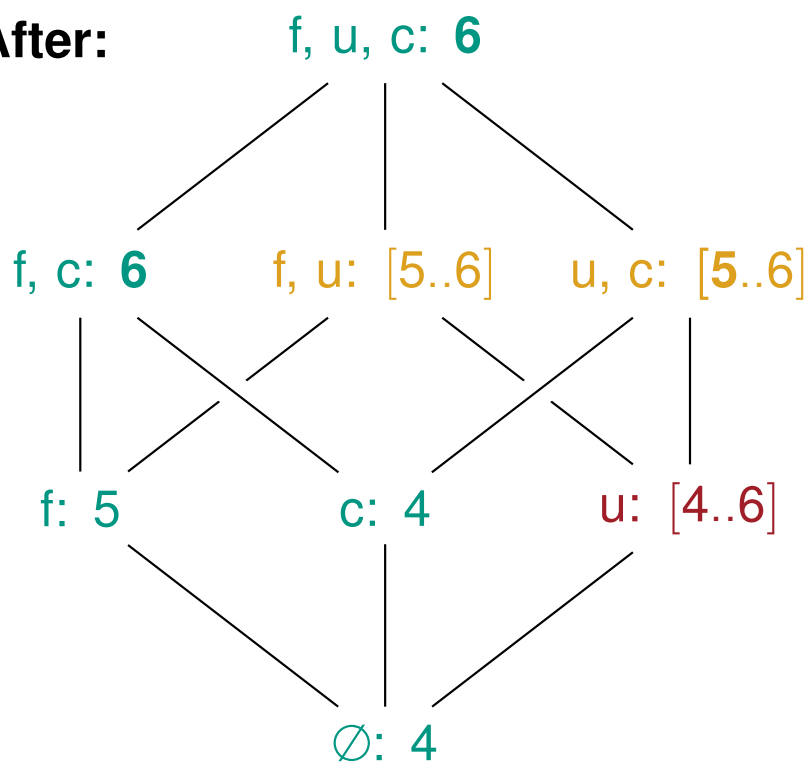
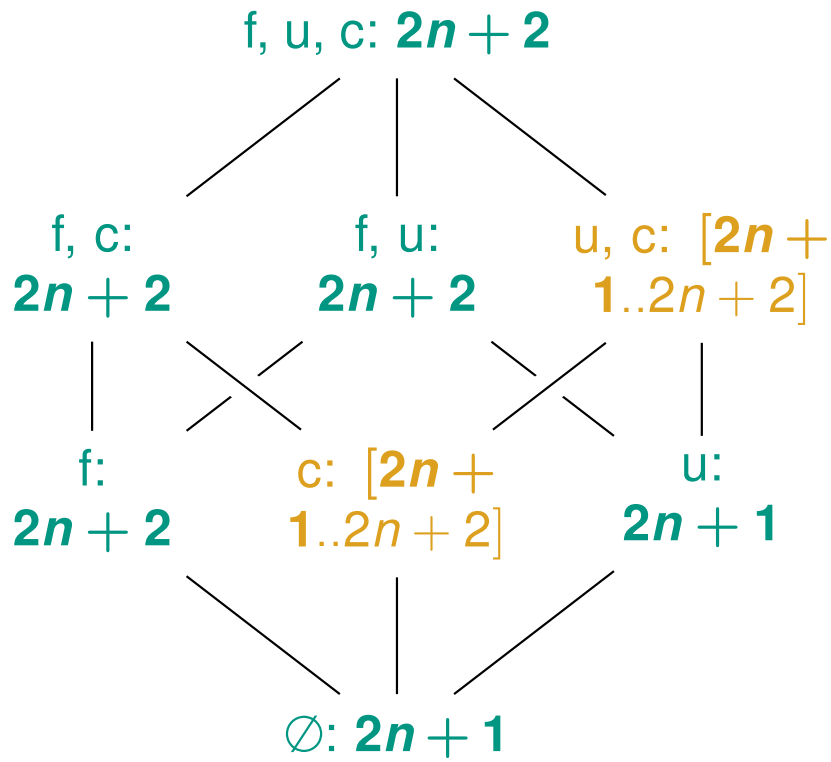
After:





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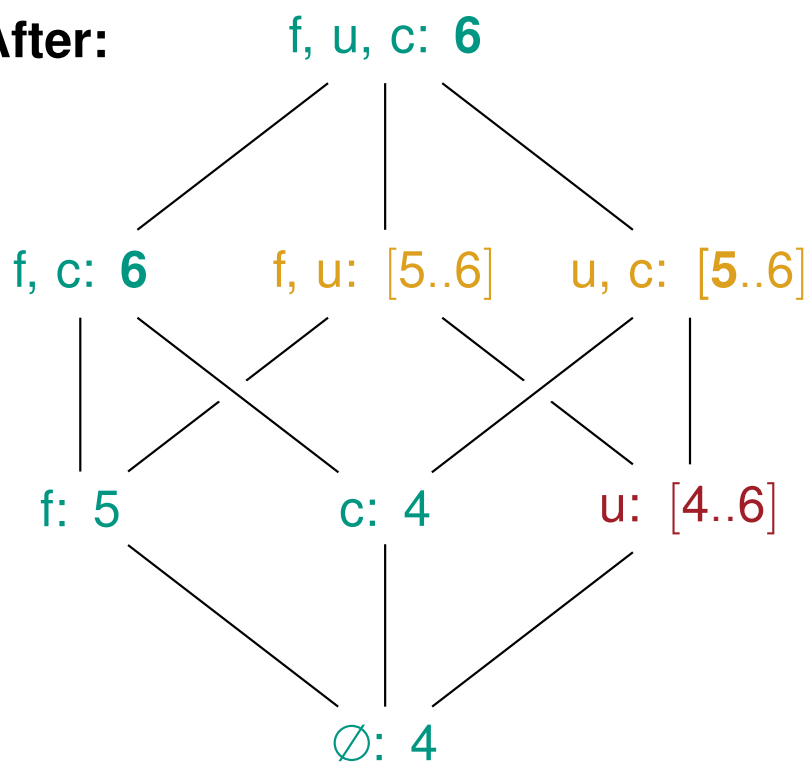
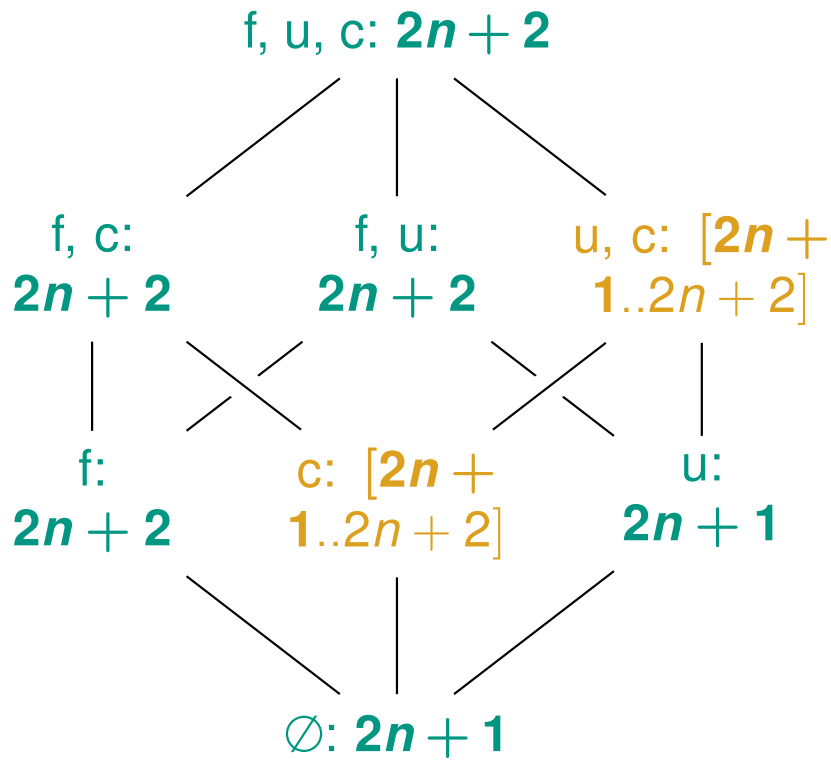
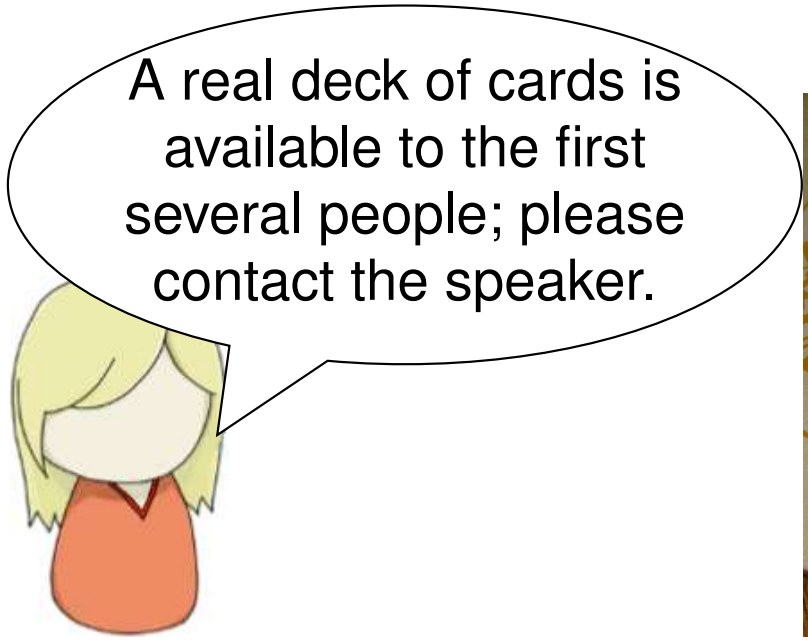




Open: 5-card AND in “f, c” if helping card is ♠?

Thank you for your attention!



References: I

A. Koch and S. Walzer. Foundations for Actively Secure Card-based Cryptography. 2017. iacr: 2017/423.

A. Koch, S. Walzer, and K. Härtel. “Card-based Cryptographic Protocols Using a Minimal Number of Cards”. In: ASIACRYPT 2015. LNCS 9452. Springer, 2015, pp. 783–807.

T. Mizuki and H. Sone. “Six-Card Secure AND and Four-Card Secure XOR”. In: FAW 2009. LNCS 5598. Springer, 2009, pp. 358–369.

A. Nishimura, T. Nishida, Y. Hayashi, T. Mizuki, and H. Sone. “Card-Based Protocols Using Unequal Division Shuffle”. In: Soft Computing (2017).